FINAL
Examination Paper

(COVER PAGE)

Session : April 2014

Programme : Diploma In Information And Communication Technology (DICTN)

Course : MAT1104 : Discrete Mathematics

Date of Examination : July 22, 2014

Time : 2:00pm – 4:00pm Reading Time: ______________

Duration : 2 Hours

Special Instructions : 

Answer any FOUR (4) structured-type questions.

Materials permitted : Non-Programmable Calculator

Materials provided : Nil

Examiner (s) : Ms. Kumatha Thinakaran, S.M. Elizabethrani

Moderator : Mr. Cheng Siak Peng

This paper consists of 7 printed pages, including the cover page.
Instruction: Answer any FOUR (4) out of FIVE (5) structured-type questions.

Question 1

(a) Rewrite each of the term of the expression

\[ 7312.56_8 - BA8.23_{16} - 157.25_{10} \]

into binary and simplify the expression. Present the final answer in hexadecimal. (6 marks)

(b) Given the following propositions:
   p: Shanon reads Readers Digest
   q: Shanon reads Cleo
   r: Shanon reads Time

Write each of the following in symbolic form:

(i) Shanon reads Readers Digest or Cleo. (2 marks)

(ii) It is not true that Shanon reads Readers Digest and Time. (2 marks)

(iii) It is not true that Shanon reads Time or Cleo but not Readers Digest. (2 marks)

(c) Calculate 2039 + 978 using 8-4-2-1 BCD. (5 marks)

(d) Convert 145.84375 to binary 32-bit floating point numbers where 8 bits are used for the characteristic and the exponent bias is \(2^7 - 1\). (8 marks)
Question 2

(a) Determine each of the following sentences is proposition. If the sentence is a proposition, write its negation.

(i) There exist integers \( x, y \) such that \( x + y = 9 \).  
(2 marks)

(ii) Leonardo da Vinci has painted the famous portrait of Mona Lisa.  
(2 marks)

(b) Let \( A = \{1,3,7,9,10\} \), and let \( R \) be the relation on \( A \) defined by ‘\( x \leq y \)’, \( R = \{(x,y) \in A : x \leq y \} \)

(i) Write \( R \) as a set of ordered pairs.  
(1 mark)

(ii) Determine whether \( R \) is reflexive, symmetric, and/or transitive.  
(2 marks)

(iii) Find the zero-one matrix for relation \( R \).  
(2 marks)

(c) (i) Simplify the following expression using the Karnaugh Map.

\[
xyzt + xyzt + x y z t + xyzt + x y z t + x y z t + x y z t
\]

(ii) Draw a logic circuit for the simplified expression.  
(8 marks)  
(3 marks)

(d) Use mathematical induction to prove that the following statement is true for any integer \( n \geq 1 \).

\[
1 + 3 + 5 + ... + (2n - 1) = n^2
\]

(5 marks)
Question 3

(a) 200 people were surveyed about their holiday destinations. The following information was obtained:

89 people want to go to Thailand (T)
73 people want to go to Singapore (S)
76 people want to go to Australia (A)
17 people want to go to both Thailand and Singapore
14 people want to go to both Thailand and Australia
20 people want to go to both Singapore and Australia
x people want to go to all the three places

(i) Given that 4 of them don’t want to go to any of the three destinations, draw a Venn diagram representing this information. (4 marks)

(ii) Find the value of x. (2 marks)

(iii) Find the number of people who want to visit Australia and Thailand, but not Singapore. (1 mark)

(b) The function \( f(x) = ax + b \). Given that \( f(-2) = 27 \) and \( f(1) = 15 \), find the value of \( x \) such that \( f(x) = -5 \). (5 marks)

(c) If numbers are held in 8 bit register in two’s complement, show how \( 75 - 110 \) would be evaluated. Justify your answer. (5 marks)

(d) Given \( p = T \), \( q = F \) and \( r = T \), find the truth value of the proposition give below. (F = false, T = true)

\( (p \land \sim q) \leftrightarrow (r \rightarrow q) \) (4 marks)

(e) Construct a truth table for the boolean expression given below.

\( f(x, y, z) = \overline{x}(yz + \overline{z}) \) (4 marks)
Question 4

(a) Form the Disjunctive Normal Form (DNF) expression from the table below. Then sketch a Karnaugh Map to simplify the Boolean Expression. Write out the simplest form.

\[
\begin{array}{ccc|c}
 x & y & z & \text{Output} \\
 0 & 0 & 0 & 0 \\
 0 & 0 & 1 & 0 \\
 0 & 1 & 0 & 0 \\
 0 & 1 & 1 & 0 \\
 1 & 0 & 0 & 1 \\
 1 & 0 & 1 & 1 \\
 1 & 1 & 0 & 1 \\
 1 & 1 & 1 & 1 \\
\end{array}
\]

(10 marks)

(b) Simplify the following equation using Boolean algebra and De Morgan’s theorems. Draw the logic circuit for the simplified equation.

\[X = \overline{AB}.(A + C)\]

(5 marks)

(c) Obtain a boolean expression for the following logical network diagram.

(d) Consider the (2, 8) encoding function \(e\),

\[
\begin{align*}
e(00) &= 00000000 \\
e(01) &= 10110100 \\
e(10) &= 01100010 \\
e(11) &= 11010111
\end{align*}
\]

(i) Find the minimum distance of the encoding function. (4 marks)

(ii) How many errors will the encoding function detect? (2 marks)
Question 5

(a) Consider the following graph. Find

(i) the number of vertices. (1 mark)
(ii) the number of edges. (1 mark)
(iii) the degree of each vertex and verify the Handshaking Theorem. (3 marks)
(iv) the number of loops. (1 mark)

(b) Use the Dijkstra’s algorithm to find the shortest path between the nodes \( a \) and \( d \) in the diagram shown below:

(4 marks)
(c) Answer the following questions for the tree.

(i) Find the ancestors of \( k \). (1 mark)

(ii) Find the children of \( d \). (1 mark)

(iii) Find the descendents of \( c \). (1 mark)

(iv) Find the siblings of \( j \). (1 mark)

(v) Draw the subtree rooted at \( e \). (1 mark)

(vi) What is the height of this rooted tree? (1 mark)

(d) Encrypt the message UPLOAD using the RSA system with \( n = 53 \times 61 \) and \( e = 17 \), translating each letter into integers and grouping together pairs of integers. (9 marks)