FINAL
Examination Paper

(COVER PAGE)

Session : January 2013

Programme : Diploma In Business (DIB)
Diploma In Information And Communication Technology (DICTN)

Course : MAT1104 : Discrete Mathematics

Date of Examination : March 4, 2013

Time : 11:00am – 1:00pm   Reading Time: 

Duration : 2 Hours

Special Instructions :

Answer any FOUR (4) structured-type questions.

Materials permitted :

Non-Programmable Calculator

Materials provided :

Nil

Examiner (s) : Mr. Aung Min, Kumatha Thinakaran, Elizabethrani Allappan.

Moderator : Dr. Ng Set Foong

This paper consists of 5 printed pages, including the cover page.
Instructions: This question paper consists of FIVE (5) questions. Answer any FOUR (4) questions in the answer booklet provided. All questions carry equal marks.

Question 1

(a) Convert the following accordingly (show all your working clearly):

(i) 10011100.1011011 binary to hexadecimal and octal (4 marks)
(ii) 4E0.6B hexadecimal to denary (3 marks)
(iii) 129.5625 denary to octal (3 marks)

(b) Find the 16-bit computer representations of the following integers:

(i) -5789 (3 marks)
(ii) 3676 (3 marks)

(c) Find the 32-bit computer representations of the decimal number "-247.125", assuming 8 bits are used for the characteristic, and the exponent bias is $2^7 - 1$. (6 marks)

(d) Perform the following calculation in BCD arithmetic:

2785+3419 (3 marks)
Question 2

(a) Prove the De Morgan’s law \((\bar{x} + y) = \bar{x} \bar{y}\) using truth-table. 

(b) Given a Boolean expression \(F = (\bar{x} + z)(y + z)\).
   
   (i) Simplify \(F\) using Boolean algebra and De Morgan’s law. 
   
   (ii) Draw a logic circuit for the original expression \(F\).

   (iii) Draw the logic circuit for the simplified expression of \(F\).

(c) Find the sum-of-products expansion of the following Boolean function:
   \(f(a, b, c) = a(b + \bar{c}) + ab + bc\)

(d) Use a Karnaugh map to find the minimal sum for the following expression:
   \(F = \bar{a}bcd + \bar{a}bc\bar{d} + abc\bar{d} + \bar{a}b\bar{c}\bar{d} + \bar{a}bcd\)

Question 3

(a) Determine the converse, contrapositive and inverse of the following statement. Which one is equivalent to the original statement?
   
   “He comes late whenever it is raining.”

(b) Use the laws of logic to classify the following expression as tautology or contradiction.
   \([s \rightarrow (r \rightarrow s)] \lor (s \land \neg r)\)

(c) Given \(p=T\), \(q=T\) and \(r=F\), find the truth value of the proposition given below.
   \((p \land q \land r) \lor (\neg r) \leftrightarrow (q \rightarrow \neg p)\)

(d) Rewrite the following statements without using the conditional:
   
   (i) If it is cold, I wear a sweater.

   (ii) Wages rise if and only if productivity increases.

(e) Prove that \(2 + 6 + 18 + \cdots + 2(3)^{n-1} = 3^n - 1\) whenever \(n\) is a nonnegative integer by using Mathematical Induction method.
Question 4

(a) Let \( A = \{1,2,5,6\} \), \( B = \{2,5,7\} \), \( C=\{1,3,5,7,9\} \) and the universal set is \( U=\{1,2,3,\ldots,8,9\} \). Find

(i) \(|A|\) (cardinality of \( A \)) and \(|B|\) (cardinality of \( B \)) \hspace{2cm} (2 \text{ marks})

(ii) \( A \cup B \) and \( B \cup C \) \hspace{2cm} (2 \text{ mark})

(iii) \( A \cap B \) and \( A \cap C \) \hspace{2cm} (2 \text{ mark})

(iv) \( A - B \) and \( A - C \) \hspace{2cm} (2 \text{ mark})

(v) \( A \oplus B \) \hspace{2cm} (2 \text{ marks})

(b) Use set builder notation and logical equivalences to establish the second De Morgan’s law
\(\overline{A \cup B} = \overline{A} \cap \overline{B} \). \hspace{2cm} (5 \text{ marks})

(c) Let \( A=\{1,2,3,4\} \), \( B=\{a,b,c\} \), \( C=\{x,y,z\} \). Consider the relation \( R \) from \( A \) to \( B \) and relation \( S \) from \( B \) to \( C \) as follows:
\[ R= \{(1,b), (3,a), (3,b), (4,c)\} \]
\[ S= \{(a,y), (c,x), (a,z)\} \]

(i) Draw the diagrams of \( R \) and \( S \). \hspace{2cm} (3 \text{ marks})

(ii) Find the matrix of each relation \( R \), \( S \) and \( R \circ S \). \hspace{2cm} (3 \text{ marks})

(iii) Write \( R^{-1} \) and \( R \circ S \) as sets of ordered pairs. \hspace{2cm} (4 \text{ marks})

Question 5

(a) Consider the following encoding function \( e: \)

\[
\begin{align*}
e(00)&=01011010 \\
e(01)&=10011010 \\
e(10)&=01010001 \\
e(11)&=11001010
\end{align*}
\]

(i) Find the minimum distance of \( e \). \hspace{2cm} (4 \text{ marks})

(ii) How many errors can \( e \) detect? \hspace{2cm} (1 \text{ mark})
(b) Encrypt the message ‘UP’ using the RSA system with \( n = 53 \cdot 61 \) and \( e = 17 \).

(5 marks)

(c) Find the length of a shortest path between \( a \) and \( z \) in the given weighted graph in Figure Q5(c).

(5 marks)

Figure Q5 (c)

(d) Consider the following graph in Figure 5 (d), find:

(i) The number of vertices

(ii) The number of edges

(iii) The number of loops

(iv) The number of pendant vertices

(v) Names of vertices which have parallel edges

(vi) The degree of each vertex and verify the Handshaking Theorem.

(4 marks)

Figure Q5 (d)

-THE END-

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