

FINAL
Examination Paper

(COVER PAGE)

Session : January 2018

Programme : Diploma In Mechanical Engineering (DMEN)

Course : **MAT1123: Engineering Mathematics 3**

Date of Examination : March 5, 2018 (Monday)

Time : 11:00 am – 1:00 pm Reading Time: Nil

Duration : 2 Hours

Special Instructions :

This paper consists of **FIVE (5)** questions. Answer any **FOUR (4)** out of **FIVE (5)** questions in the answer booklet provided. All questions carry equal marks.

Materials permitted :
Non-Programmable Scientific Calculator

Materials provided :
Formula Booklet 1

Examiner (s) : **Dr Chan Kah Yein & Bark Chee Beg**

Moderator : Mr Foo Kim Eng

This paper consists of 4 printed pages, including the cover page.

DIPLOMA IN MECHANICAL ENGINEERING PROGRAMME (DMEN)
MAT1123 : ENGINEERING MATHEMATICS 3
FINAL EXAMINATION : JANUARY 2018 SESSION

Instructions: This paper consists of **FIVE (5)** questions. Answer only **FOUR (4)** questions in the answer booklet provided. All questions carry equal marks. Working must be shown.

Question 1

- (a) Use elementary row operations to solve the system below. Leave your solution in vector form.

$$\begin{aligned}x_1 + 3x_2 - 4x_3 + 4x_4 &= 4 \\x_1 + 4x_2 - 7x_3 + 6x_4 &= 3\end{aligned}\quad (8 \text{ marks})$$

- (b) Let matrix $\mathbf{A} = \begin{pmatrix} 1 & 3 \\ 4 & -3 \end{pmatrix}$.

- (i) Find the eigenvalues of \mathbf{A} . (4 marks)

- (ii) Find a set of linearly independent eigenvectors for \mathbf{A} . (7 marks)

- (c) Consider the following system of linear equations:

$$\begin{aligned}2x - 11y - z &= 36 \\8x - y + 2z &= 21 \\x - y + 9z &= 14\end{aligned}$$

- (i) Set up a Gauss-Seidel scheme for the system. (3 marks)

- (ii) Compute one (1) iteration, starting with initial guess $x = y = z = 1$. Keep 3 decimal places in all calculations. (3 marks)

Question 2

- (a) Let the points $A(1, -1, 4)$, $B(2, 1, -1)$ and $C(3, 1, 3)$. Find the following:
- (i) vectors \overrightarrow{AB} and \overrightarrow{AC} . (2 marks)
 - (ii) a vector orthogonal to \overrightarrow{AB} and \overrightarrow{AC} . (4 marks)
 - (iii) the equation of the plane consisting of the points A, B and C. (5 marks)
- (b) Let the line integral $I = \int_C [(12x^2 + 3y^2 + 5y)dx + (6xy - 3y^2 + 5x)dy]$.
- (i) Show that I is independent of path. (5 marks)
 - (ii) Find the scalar potential and hence, evaluate I if C is a simple path joining the points $(0, 0)$ and $(2, 1)$ in the xy -plane. (9 marks)

Question 3

- (a) Evaluate the line integral $\oint_C [(xy) dx + (x^2 - y^2) dy]$ where C is the triangular path OAB defined by $O(0,0)$, $A(1,0)$ and $B(1,1)$ in the counter-clockwise direction, using
- (i) Green's theorem, (8 marks)
 - (ii) line integration. (11 marks)
- (b) Given that $f = x^2 + 2xy^2 + 2yz^3$ and $\vec{v} = 4\hat{i} - 2\hat{j} - 2\hat{k}$, find the directional derivative of f in the direction of \vec{v} at the point $(2, 1, -1)$. (6 marks)

Question 4

- (a) Use Stokes' theorem to evaluate $\oint_C \vec{F} \cdot d\vec{r}$ where $\vec{F} = (y)\hat{i} + (xz)\hat{j} + \hat{k}$ and C is the boundary of the circle $x^2 + y^2 = 1$ in a counter-clockwise orientation. (10 marks)
- (b) The position vector of a particle is $\vec{r}(t) = (\sin 2t)\hat{i} + (\cos t)\hat{j} + (3t)\hat{k}$, where $t \geq 0$ is time. Let Q be the point $(0, -1, 3\pi)$.
- (i) Find the value of t at point Q . (2 marks)
- (ii) Find the velocity vector \vec{v} and acceleration vector \vec{a} at point Q . (8 marks)
- (c) Let the vectors $\vec{u} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{w} = 2\hat{i} + 3\hat{j} - 6\hat{k}$. Find the angle between the two vectors. Leave your answer in 1 decimal place. (5 marks)

Question 5

- (a) Use Gauss' divergence theorem to evaluate $\iiint_S \vec{F} \cdot d\vec{S}$, where $\vec{F} = (2x)\hat{i} + (y^2)\hat{j} + (z)\hat{k}$ and S is the surface of the solid completely enclosed by the cylinder $x^2 + y^2 = 9$ and the planes $z = 0$ and $z = 5$. (16 marks)
- (b) Set up double integrals with correct limits to evaluate the area of following regions. Do not integrate.
- (i) The region bounded by the circle $x^2 + y^2 = 7$ and the lines $y = x$ and $y = \sqrt{3}x$ in the first quadrant. (4 marks)
- (ii) The region bounded by the curve $y = x^2$ and the lines $5x + y = 6$ and $x = 0$ in the first quadrant. (5 marks)

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