

FINAL
Examination Paper

(COVER PAGE)

Session : August 2019

Programme : Diploma In Mechanical Engineering (DMEN)

Course : MAT1123 : Engineering Mathematics 3

Date of Examination : December 12, 2019 (Thursday)

Time : 5:00 pm – 7:00 pm Reading Time: Nil

Duration : 2 Hours

Special Instructions :

This paper consists of **FIVE (5)** questions. Answer any **FOUR (4)** out of **FIVE (5)** questions in the answer booklet provided. All questions carry equal marks.

Materials permitted : Non-Programmable Scientific Calculator

Materials provided : Formula Booklet 1

Examiner (s) : Dr Chan Kah Yein and Dinash a/l Kandasamy

Moderator : Assoc Prof Chan Kait Loon

This paper consists of 3 printed pages, including the cover page.

DIPLOMA IN MECHANICAL ENGINEERING PROGRAMME (DMEN)
 MAT1123 : ENGINEERING MATHEMATICS 3
 FINAL EXAMINATION : AUGUST 2019 SESSION

Instructions: This paper consists of **FIVE (5)** questions. Answer only **FOUR (4)** questions in the answer booklet provided. All questions carry equal marks. Working must be shown.

Question 1

- (a) Use elementary row operations to find the solution to the system below. Give your answer in vector form.

$$\begin{aligned}x_1 + 2x_2 + 3x_3 &= 4 \\5x_1 + 6x_2 + 7x_3 &= 8 \\9x_1 + 10x_2 + 11x_3 &= 12\end{aligned}\quad (8 \text{ marks})$$

- (b) Use Cramer's Rule to solve for z in the system below.

$$\begin{aligned}2x - 4y + z &= 12 \\x + 3y - 2z &= 1 \\3x + y - 4z &= 19\end{aligned}\quad (6 \text{ marks})$$

- (c) Let matrix $\mathbf{A} = \begin{pmatrix} 2 & -4 \\ -3 & 1 \end{pmatrix}$.

- (i) Find the eigenvalues of \mathbf{A} . (4 marks)
- (ii) Find a set of linearly independent eigenvectors for \mathbf{A} . (7 marks)

Question 2

- (a) Let the points $A(1, 0, -2)$, $B(2, 1, 3)$ and $C(-1, 3, 0)$. Find the following:

- (i) vectors \overrightarrow{AB} and \overrightarrow{AC} . (2 marks)
- (ii) a vector orthogonal to \overrightarrow{AB} and \overrightarrow{AC} . (4 marks)
- (iii) the equation of the plane consisting of the points A , B and C . (5 marks)

- (b) Let the vectors $\vec{u} = 4\hat{i} - 2\hat{j} - 3\hat{k}$ and $\vec{v} = -\hat{i} + 2\hat{j} + 3\hat{k}$. Find the cosine of the angle between the two vectors. Leave your answer in surds. (5 marks)

- (c) Set up double integrals with the correct limits to evaluate the area of following regions. Do not integrate.
- (i) The region bounded by the circle $x^2 + y^2 = 9$ and the lines $y = x$ and the x-axis in the first quadrant. (4 marks)
- (ii) The region bounded by the curve $y = x^2$ and the lines $y = 8 + 2x$ and $x = 0$ in the first quadrant. (5 marks)

Question 3

Find $\oint_C \vec{F} \cdot d\vec{r}$ where $\vec{F} = (-y)\hat{i} + (x)\hat{j} + (z)\hat{k}$ and C is the rectangle described by $0 \leq x \leq 4$, $0 \leq y \leq 2$, $z = 0$. Assume a counter-clockwise orientation for the path, as viewed from above. Use

- (a) line integration, (15 marks)
- (b) Stokes' Theorem. (10 marks)

Question 4

- (a) Use Green's Theorem to find the work done by the force $\vec{F} = (e^x - y^3)\hat{i} + (\sin y + x^3)\hat{j}$ on a particle that travels once around the circle $x^2 + y^2 = 4$ in a counterclockwise direction. (15 marks)
- (b) Given that $f = x^2y^2 + 3y^2z - 4xz^2$ and vector $\vec{v} = 4\hat{i} - 3\hat{j} + 4\hat{k}$, find the directional derivative of f in the direction of \vec{v} at the point $A(3, 2, -1)$. (10 marks)

Question 5

- (a) Use Gauss' Divergence Theorem to evaluate $\oiint_S \vec{F} \cdot d\vec{S}$ where $\vec{F} = (xy)\hat{i} - (\frac{1}{2}y^2)\hat{j} + (z^2)\hat{k}$ and S is the closed surface of the solid bounded by the cylinder $x^2 + y^2 = 9$ and the planes $z = 0$, $z = 5$. (15 marks)
- (b) The position vector of a particle is $\vec{r}(t) = (2t^2)\hat{i} + (\sin 2t)\hat{j} + (\cos 3t)\hat{k}$, where $t \geq 0$ is time. Let Q be the point $(2\pi^2, 0, 1)$.
- (i) Find the value of t at point Q. (2 marks)
- (ii) Find the velocity vector \vec{v} and acceleration vector \vec{a} at point Q. (8 marks)

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