INTI
International College Penang
LAUREATE INTERNATIONAL UNIVERSITIES

FINAL /RESIT
Examination Paper

(COVER PAGE)

Session	:	AUGUST 2012
Programme	:	DIPLOMA IN INFORMATION AND COMMUNICATION TECHNOLOGY PROGRAMME (DICTN)
Course	:	**MAT1104 : DISCRETE MATHEMATICS**
Date of Examination	:	11 DECEMBER 2012
Time	:	2p.m. – 4p.m. Reading Time : Nil
Duration	:	2 Hours
Special Instructions	:	

This paper consists of **FIVE (5)** questions. Answer any **FOUR (4)** questions in the answer booklet provided. All questions carry equal marks.

Materials permitted :

Non-programmable calculator

Materials provided :

Nil

Examiner(s) : **Chan Ah Wah**

Moderator : **Kumatha**

This paper consists of 6 printed pages, including the cover page.

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MAT1104 DISCRETE MATHEMATICS

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Instructions

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Question 1

(a) Write down the expanded forms of 2386.75_{10} and 1101.01_2 .

[4 marks]

(b) Convert 25.32_{10} to its binary representation with 5 digits after the binary point.

[8 marks]

(c) Convert 275.4375_{10} to its octal representation.

[5 marks]

(d) Convert 985.78125_{10} to its hex representation.

[5 marks]

(e) Convert 10100011.10111_2 to its octal representation.

[3 marks]

Question 2

(a) Compute $11010_2 \times 1011_2$ by using long multiplication method.

[3 marks]

(b) Find the 2's complement of $1101\ 0100_2$.

[2 marks]

(c) Determine the 10's complement of 3269_{10} .

[2 marks]

(d) Show how the following operations are carried out in a 4-bit computer :

(i) $2+3$

[3 marks]

(ii) $7-4$

[3 marks]

(e) A real number x is in normalised binary exponential form if it is written as

$$x = \pm m \times 2^e$$

where the significand m lies in the range $0.1_2 \le m \le 1_2$ and the exponent e is an integer written in its decimal representation. Express the following numbers in normalised binary exponential form and, hence, determine the values of m and e in each case :

(i) 11001.101_2 [3 marks]

(ii) 0.000110111_2 [3 marks]

(f) Find the 32-bit computer representation of

$$x = -0.1001\ 1110_2 \times 2^{-4}$$

where the first bit is the sign bit, and the following 8 bits and 23 bits are used for the characteristic and the significand respectively. Assume that the exponent bias is $2^7 - 1$.

[6 marks]

Question 3

(a) (i) Define the term 'proposition'. [1 mark]

(ii) State the converse of $p \to q$. [1 mark]

(iii) State the contrapositive of $p \to q$. [1 mark]

(b) Express the following propositions in symbolic form (i.e. in terms of p, q, r), and identify the principal connective in each case.

(i) Either Sarah is studying computing and Peter is not studying mathematics, or Peter is studying mathematics. [2 marks]

(ii) It is not the case that if it is sunny then I will carry an umbrella. [2 marks]

(iii) The program will terminate if and only if the input is not numeric or the escape key is pressed. [2 marks]

(c) Write down in English sentences the **converse** and **contrapositive** of the following propositions:

(i) If the input file exists, then an error message is not generated. [2 marks]

(ii) If the database is not accessible, then my program cannot run.

[2 marks]

(iii) If my program contains no bugs, then it produces correct output.

[2 marks]

(d) (i) Use a truth table to verify the first de Morgan's law : $\neg(p \wedge q) \equiv \neg p \vee \neg q$.

[5 marks]

(ii) Determine a truth table for $\neg p \rightarrow (q \rightarrow p)$.

[3 marks]

(iii) Show that $(p \wedge q) \rightarrow p$ is a tautology.

[2 marks]

Question 4

(a) Let

$$\mathrm{E} = \{x \in \mathrm{N} : x \leq 12\} \text{ [Universal Set]},$$

$$\mathrm{A} = \{x : x \text{ is odd}\},$$

$$\mathrm{B} = \{x : x > 7\}, \text{ and}$$

$$\mathrm{C} = \{x : x \text{ is divisible by } 3\}.$$

Write down the following sets in enumerated form :

(i) $\mathrm{B} \cup \mathrm{C}$

(ii) $\overline{\mathrm{A}}$

(iii) $(\mathrm{A} \cup \overline{\mathrm{B}}) \cap \mathrm{C}$

(iv) $\overline{\mathrm{A} \cup \mathrm{C}} \cup \overline{\mathrm{C}}$

[4 marks]

(b) Let $\mathrm{A} = \{1, 2, 3, 4, 5\}$, and let R be the relation on A defined as follows:

$$\mathrm{R} = \left\{(1,3), (1,4), (2,1), (2,2), (2,4), (3,5), (5,2), (5,5)\right\}$$

(i) Write down the matrix representation of R.

(ii) Draw the graphical representation of R.

[5 marks]

(c) Let the universal set be $\mathrm{E} = \left\{1, 2, 3, ..., 10\right\}$.

(i) Find the representation of $\{2, 3, 5, 7\}$ as a bit string.

[2 marks]

(ii) Find the set represented by the bit string 1001 0110 11.

[2 marks]

In each case, describe briefly how you arrive at the answer.

(d) Let the functions f, g, h be defined as follows :

$$f : \mathbb{R} \to \mathbb{R},\ f(x) = 4x - 3$$

$$g : \mathbb{R} \to \mathbb{R},\ g(x) = x^2 + 1$$

$$h : \mathbb{R} \to \mathbb{R},\ h(x) = \begin{cases} 1 & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases}$$

Find rules for the following functions with domain and codomain $\mathbb{R}$:

(i) $f \circ g$

[1 mark]

(ii) $h \circ f$

[2 marks]

(iii) $g \circ h$

[2 marks]

(e) Consider the recurrence relation $a_n = a_{n-1} + 2a_{n-2}$ with initial conditions $a_0 = 0,\ a_1 = 7$.

(i) Find the general solution.

[3 marks]

(ii) Find the unique solution with the given initial conditions.

[4 marks]

Question 5

(a) Determine the Boolean function $f(x, y, z)$ given by the following table. Leave your answer in disjunctive normal form.

x	y	z	$f(x,y,z)$
1	1	1	0
1	1	0	1
1	0	1	1
1	0	0	0
0	1	1	0
0	1	0	1
0	0	1	1
0	0	0	0

[2 marks]

(b) Use a Karnaugh map to simplify the Boolean expression

$$xy'z + xyz' + xy'z' + x'y'z + x'y'z'.$$

[5 marks]

(c) Draw a circuit to implement the expression $xy' + (xy)'$ exactly as it is written. Then try to find a simpler circuit to implement the expression.

[4 marks]

(d) Use the laws of Boolean algebra to simplify the expression $(x + yz)(x' + z)$.

[4 marks]

(e) (i) Define the terms *Eulerian path* and *Eulerian circuit*.

[2 marks]

(ii) Determine the Eulerian circuit of Figure Q5(e) below, assuming that you start from vertex 1.

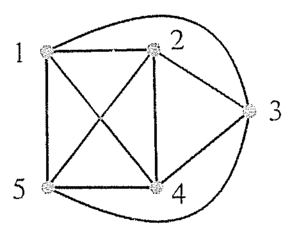

Figure Q5(e)

[2 marks]

(f) Prove by induction that the following statement is true for all natural numbers n :

$$1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{1}{6}n(n+1)(2n+1)$$

[6 marks]

—— End of Paper ——