INTI
INTERNATIONAL COLLEGE PENANG (507232-U)
LAUREATE INTERNATIONAL UNIVERSITIES

FINAL
Examination Paper

(COVER PAGE)

Session : JAN 2015

Programme : DIPLOMA IN ELECTRICAL AND ELECTRONIC ENGINEERING

Course : MAT1122 : ENGINEERING MATHEMATICS II

Date of Examination : 14 March 2015 (Saturday)

Time : 8:00 am – 10:00 pm Reading Time : Nil

Duration : 2 Hours

Special Instructions :

This paper consists of **FIVE1 (5)** questions. Answer any **FOUR (4)** questions in the answer booklet provided. All questions carry equal marks.

Materials permitted :

Non-Programmable Scientific Calculator

Materials provided :
Graph paper with diode characteristic (for Question 3(c))

Examiner(s) : **Lee Chin-Aik**

Moderator : **Dr. Ch'ng Pei Eng**

This paper consists of 4 printed pages, including the cover page.

INTI INTERNATIONAL COLLEGE PENANG

DIPLOMA IN ELECTRICAL AND ELECTRONIC ENGINEERING (DEEI)
MAT1122 ENGINEERING MATHEMATICS 2
FINAL EXAMINATION: JANUARY 2015 SESSION

Instructions: This paper consists of **FIVE (5)** questions. Answer any **FOUR (4)** questions in the answer booklet provided. All questions carry equal marks.

Question 1

The following gambling game is being played. There are four chips in an urn: a red chip, a yellow chip and two blue chips. Whenever the yellow chip is chosen, no monetary exchange is made. Whenever a blue chip is chosen, the gambler is given RM2.00. And whenever the red chip is chosen, the gambler has to pay the dealer RM4.50. Each round, the gambler picks a chip randomly from the urn, and then returns it back into the urn.

(a) What is the mode chip color? [1 mark]

(b) Find the mean and standard deviation of the money gained per round. [3 marks]

(c) Suppose this game continues over 1000 rounds. What is the probability that the gambler *doesn't* lose any money by the end of the game? [3 marks]

(d) Suppose signal glitches occur uniformly randomly over time along a communication channel at the average rate of 5.5 glitches per hour. Over a 30 minute period, what is the probability that four or more glitches will occur? [3 marks]

(e) An alternating current circuit has the potential difference $V(t) = V_0 \cos(\omega t)$ and current $I(t) = \dfrac{V_0}{R}\cos(\omega t - \varphi)$. The power consumed by the circuit, $P = VI$. Compute the rate of change of the power with respect to time. [5 marks]

(f) Compute the average power over a cycle, $\overline{P} \equiv \dfrac{\omega}{2\pi}\displaystyle\int_0^{2\pi/\omega} P(t)\,dt$. [5 marks]

(g) Suppose $V_0 = 100V \pm 1\%$, $R = 1000\Omega \pm 2\%$ and $\varphi = \frac{\pi}{6}\text{rad} \pm 5\%$. What is the margin of error of the average power in Watts? [5 marks]

Question 2

Let $z = (5 + i)^4(239 - i)$.

(a) Using de Moivre's formula, express $\arg z$ in the form $n_1 \arctan\left(\frac{p_1}{q_1}\right) + n_2 \arctan\left(\frac{p_2}{q_2}\right)$ for some integers $n_1, n_2, p_1, p_2, q_1, q_2$. [5 marks]

(b) Express z in Cartesian form. [5 marks]

(c) Use the result of part (b) to verify that $\arg z = \frac{\pi}{4}$. [2 marks]

(d) Using the power series expansion $\arctan x = \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} x^{2n+1}$ up to order x^7 together with the results of parts (a) and (c), evaluate π *correct* to *five* decimal places using *only* arithmetic operations on rational numbers. [5 marks]

(e) The exact solution to the differential equation $f' = f$ with the initial condition $f(0) = 1$ is given by $f(x) = e^x$. However, it can also be approximated numerically using the Euler method with n steps. Using this approximation, show that $e = f(1) \approx \left(1 + \frac{1}{n}\right)^n$ for large n. (Hint: use iteration to figure out the approximate value of f on the r^{th} step.) [6 marks]

(f) Evaluate e to 6 decimal places using this approximation with $n = 10^7$. [2 marks]

Question 3

Evaluate the following integrals.

(a) $\int \cos(\ln x)\, dx$, $x > 0$ (Hint: integrate by parts.) [6 marks]

(b) $\int_{-1}^{1} x\sqrt{\frac{1+x}{1-x}}\, dx$ (Hint: multiply the integrand by $\sqrt{\frac{f}{f}}$ for some function

f to remove the radical from the numerator.) [7 marks]

(c) $\int \frac{x^{3/4}}{x-1}\,dx,\ x \geqslant 0$ (Hint: use substitution followed by partial fraction decomposition.) [7 marks]

(d) Find the four roots of $z^4 + 4 = 0$. [5 marks]

Question 4

Given the following system of coupled second order linear differential equations

$$f'' + 2f' + 5f = g, \qquad f(0) = 1,\ f'(0) = 0$$
$$g'' + 2g' + 5g = 0, \qquad g(0) = 0,\ g'(0) = 1,$$

(a) Solve for $g(t)$. [10 marks]

(b) Using the result from part (a), convert the first differential equation into an algebraic equation using Laplace transforms, and then solve for $\mathcal{L}f$. [10 marks]

(c) Obtain $f(t)$ from the inverse Laplace transform of the previous answer. [5 marks]

Question 5

(a) Find the general solution to the linear equation $\sin x\, f' - \tan x\, f = \sec x$. [13 marks]

(b) The solution curves to the homogeneous differential equation $(y + x)\,dx + (y - x)\,dy = 0$ are logarithmic spirals. Find the solution curve to this equation passing through the point $(x, y) = (1, 0)$. [12 marks]

~ The end ~