INTI
International College Penang
LAUREATE INTERNATIONAL UNIVERSITIES

FINAL
Examination Paper

(COVER PAGE)

Session : JANUARY 2013

Programme : DIPLOMA IN ELECTRICAL & ELECTRONIC ENGINEERING (DEEI)

Course : **MAT 1123: Engineering Mathematics 3**

Date of Examination : 8 March 2013

Time : 11a.m. – 1p.m. Reading Time : Nil

Duration : 2 Hours

Special Instructions :

This paper consists of **FIVE (5)** questions. Answer any **FOUR (4)** questions in the answer booklet provided. All questions carry equal marks.

Materials permitted :

Non-programmable calculator

Materials provided :

Formula Booklet 1

Examiner(s) : **Chan Ah Wah**

Moderator : **Dr. Ch'ng Pei Cheng**

This paper consists of 4 printed pages, including the cover page.

INTI INTERNATIONAL COLLEGE PENANG

DIPLOMA IN ELECTRICAL AND ELECTRONIC ENGINEERING (DEEI)

MAT1123 ENGINEERING MATHEMATICS 3

FINAL EXAMINATION : JANUARY 2013 SESSION

Instructions

This paper consists of **FIVE (5)** questions. Answer any **FOUR (4)** questions in the answer booklet provided. All questions carry equal marks.

Question 1

(a) Explain briefly how you can use **rank test** to determine the nature of a system of equations given by $\mathbf{Ax} = \mathbf{b}$ where there are m linear equations and n unknowns.

[3 marks]

(b) Given the following system of linear equations

$$\begin{aligned} x + 4y + 5z &= -7 \\ 2x + 3y \quad\;\; &= 1 \\ x + 2y + \ z &= k \end{aligned}$$

Determine all possible values of k so that the system will have

(i) infinitely many solutions

(ii) no solution.

[6 marks]

(c) Given that $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 3$, find $\begin{vmatrix} -3a & b & c \\ -3d & e & f \\ -3g + 12d & h - 4e & i - 4f \end{vmatrix}$ using elementary row operations.

[8 marks]

(d) Find $\mathbf{A}^{-1}$ by using elementary row operations on the following matrix :

$$\mathbf{A} = \begin{bmatrix} 1 & 3 & -1 \\ -1 & 0 & 0 \\ 0 & 2 & -1 \end{bmatrix}$$

[8 marks]

Question 2

(a) Evaluate adj (P) if the matrix **P** is given by

$$\mathbf{P} = \begin{bmatrix} 1 & 4 & 3 \\ 6 & 2 & 5 \\ 1 & 7 & 0 \end{bmatrix}.$$

Show all your workings .

[7 marks]

(b) Let $\mathbf{A} = \begin{bmatrix} 1 & -1 & 8 \\ 0 & 0 & 6 \\ 0 & -1 & 5 \end{bmatrix}$.

(i) Find all the eigenvalues of **A**.

[4 marks]

(ii) Find an eigenvector corresponding to the smallest eigenvalue.

[7 marks]

(c) Solve the following system of linear equations using Gauss-Seidel method. Compute two (2) iterations, starting with the initial guess $x_1^{(0)} = 1,\ x_2^{(0)} = 1, x_3^{(0)} = 1$. Keep 4 decimal places in all calculations .

$$\begin{aligned} 8x_1 - x_2 + 2x_3 &= 21 \\ 2x_1 - 11x_2 - x_3 &= 36 \\ x_1 - x_2 + 9x_3 &= 14 \end{aligned}$$

[7 marks]

Question 3

(a) Given the scalar field $\phi(x, y, z) = x^2y^2z + yz^3$ find the directional derivative of ϕ at the point $(1, -1, -1)$ in the direction of the vector $\mathbf{A} = \mathbf{i} + 2\mathbf{j} + \mathbf{k}$.

[6 marks]

(b) Given $\mathbf{F} = (x + y)\mathbf{i} - 2y\mathbf{j}$, evaluate $\int_C \mathbf{F} \bullet d\mathbf{r}$ from $(0,1)$ to $(1,2)$ along

(i) the straight line from $(0, 1)$ to $(1,2)$

(ii) the parabola $x = t,\ y = t^2 + 1$.

[9 marks]

(c) Use the Divergence theorem to evaluate $\iint_S \mathbf{F} \bullet \mathbf{n}\, dS$ where $\mathbf{F} = xy\mathbf{i} - xz\mathbf{j} + 3yz\mathbf{k}$ and S is the closed surface of the region bounded by the planes $z = 0$, $z = 4 - y$, $y = 0$, $y = 2$, $x = 0$, and $x = 3$.

[10 marks]

Question 4

(a) Given that $F = y^3\mathbf{i} + (3xy^2 + z^2)\mathbf{j} + 2yz\mathbf{k}$. Show that $\mathbf{F}$ is a conservative vector field .

[4 marks]

(b) Use Green's theorem to solve

$$\oint_C (y^2 - 2xy)\, dx + (x + 2xy)\, dy$$

where C is the counter clockwise oriented boundary of the region R bounded by the lines $x = 0$, $y = x$, and $y = 1$. Hint : convert the line integral to a double integral .

[10 marks]

(c) A multiple integral is given by

$$\mathrm{I} = \int_0^1 \int_{x^2}^{2-x} f(x, y)\, dydx .$$

(i) Sketch and label the region of integration .

[4 marks]

(ii) Reverse the order of integration and express I as a sum of two double integrals .

[7 marks]

Question 5

(a) Let $\phi = xy^3 + yz^2$ be a scalar field . Find $\nabla \bullet (\nabla\phi)$ at the point $(1,\ 2,\ 1)$.

[5 marks]

(b) Use Stokes' theorem to evaluate $\iint_S \nabla \times \mathbf{F} \bullet \mathbf{n}\, dS$ for the function

$$\mathbf{F} = -y\,\mathbf{i} + x\,\mathbf{j} + z\,\mathbf{k}$$

where S is the hemisphere $S:\ x^2 + y^2 + z^2 = 4,\ \ z \geq 0$ and $\mathbf{n}$ is the outward pointing unit normal vector to the surface S.

[7 marks]

(c) Sketch the graph of the periodic function $f(x)$ for $-5\pi < x < 5\pi$ and expand it in a Fourier series .

$$f(x) = x + \pi, \quad -\pi < x < \pi$$
$$f(x) = f(x + 2\pi)$$

[13 marks]

—— End of Paper ——