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INTERNATIONAL COLLEGE PENANG (507232-U)
LAUREATE INTERNATIONAL UNIVERSITIES

FINAL
Examination Paper

(COVER PAGE)

Session	:	January 2012
Programme	:	Diploma in Electrical and Electronic Engineering (DEE)
Course	:	**MAT 1123: Engineering Mathematics 3**
Date of Examination	:	8 March 2012
Time	:	11a.m. – 1p.m. Reading Time : Nil
Duration	:	2 Hours
Special Instructions	:	

This paper consists of **FIVE (5)** questions. Answer any **FOUR (4)** questions in the answer booklet provided. All questions carry equal marks.

Materials permitted :

Non-programmable calculator

Materials provided :

Formula Booklet 1

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Moderator	:	**Goh Chok Huat**

This paper consists of 5 printed pages, including the cover page.

INTI INTERNATIONAL COLLEGE PENANG

DIPLOMA IN ELECTRICAL AND ELECTRONIC ENGINEERING (DEE)

MAT1123 ENGINEERING MATHEMATICS 3

FINAL EXAMINATION : JANUARY 2012 SESSION

Instructions

This paper consists of **FIVE (5)** questions. Answer any **FOUR (4)** questions in the answer booklet provided. All questions carry equal marks.

Question 1

(a) Given the following system of linear equations :

$$\begin{aligned} x + 2y + 3z &= 0 \\ x + 3y + 5z &= 1 \\ 2x + 3y + 4z &= -1 \end{aligned}$$

(i) Determine the reduced row echelon form (rref) of the system.

(ii) Use Rank Test to prove that it is consistent with infinitely many solutions.

(iii) Solve the system and express the solution in vector form.

[10 marks]

(b) Given a homogeneous system as follows :

$$\begin{aligned} x + 2y + kz &= 0 \\ y + 2z &= 0 \\ x \quad - z &= 0 \end{aligned}$$

Find the value of k if the homogeneous system has non-trivial solutions.

[6 marks]

(c) Given a 2×2 matrix $\mathbf{A} = \begin{bmatrix} 5 & 4 \\ 3 & 1 \end{bmatrix}$.

(i) Find the eigenvalues of **A**.

[3 marks]

(ii) Find an eigenvector corresponding to each of the eigenvalues.

[6 marks]

Question 2

(a) Let A be a 2×2 matrix. If det(A)= 5, find $\det(3A^{-1})$ where A^{-1} is the inverse of A.

[4 marks]

(b) Find A^{-1} using elementary row operations if matrix A is given by

$$A = \begin{bmatrix} 1 & 3 & -1 \\ -1 & 0 & 0 \\ 0 & 2 & -1 \end{bmatrix}$$

[8 marks]

(c) Determine adj(A) if matrix A is given by

$$A = \begin{bmatrix} 1 & 0 & -2 \\ -3 & 1 & 4 \\ 2 & -3 & 4 \end{bmatrix}.$$

Show all your workings.

[6 marks]

(d) Rearrange the equations (if necessary) and apply Gauss-Seidel scheme to solve the following system. Complete three (3) iterations starting with the initial guess of $x_1^{(0)} = 1$, $x_2^{(0)} = 1$, $x_3^{(0)} = 1$. Maintain 4 decimal places throughout the calculations.

$$\begin{aligned} -x_1 + x_2 + 8x_3 &= 33 \\ 10x_1 - x_2 + x_3 &= 21 \\ x_1 + 9x_2 - x_3 &= 25 \end{aligned}$$

[7 marks]

Question 3

(a) Let $\mathbf{A} = \mathbf{i} + 2\mathbf{j} + \mathbf{k}$ and $\mathbf{B} = 2\mathbf{i} - \mathbf{j} - \mathbf{k}$. Find a vector of magnitude 5 that is perpendicular to both A and B.

[5 marks]

(b) Let $\mathbf{A} = xy^3z\mathbf{i} - x^2y\mathbf{j} + zy^3\mathbf{k}$ be a vector field. Find $\nabla(\nabla \cdot \mathbf{A})$ at the point $(1,1,1)$.

[5 marks]

(c) Given a scalar field $\phi(x,y,z) = xy^2z + x^2z^3$. Find the directional derivative of ϕ at the point $(1,1,1)$ in the direction of the vector $\mathbf{A} = \mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$.

[6 marks]

(d) Use Gauss' Divergence Theorem to evaluate $\iint_S \mathbf{F} \cdot d\mathbf{S}$, where

$$\mathbf{F} = 3xyz\mathbf{i} + 4y^2z\mathbf{j} + 2yz^2\mathbf{k}$$

and S is the closed surface of a cuboid enclosed by the planes $x = 0$, $x = 3$, $y = 0$, $y = 2$, $z = 0$, $z = 1$.

[9 marks]

Question 4

(a) A force $\mathbf{F} = 2z\mathbf{i} + 5x\mathbf{j} - 4y\mathbf{k}$ displaces a particle in space from $(0,0,0)$ to $(2,1,3)$ along the curve $C: \ x = 2t, \ y = t^2, \ z = 3t$. Find the work done by the force given by the integral $\int_C \mathbf{F} \cdot d\mathbf{r}$.

[7 marks]

(b) Given $\mathbf{F} = (yz + y)\mathbf{i} + (xz + x + z)\mathbf{j} + (xy + y)\mathbf{k}$.

(i) Show that $\mathbf{F}$ is a conservative vector field.

[4 marks]

(ii) Find the scalar potential of $\mathbf{F}$.

[5 marks]

(c) Let $S: x^2 + y^2 + z^2 = 4$ be a hemisphere above the xy-plane. Suppose a force $\mathbf{F} = 2\mathbf{k}$ acts on the surface and around its boundary. Evaluate the double integral

$$\iint_S \mathbf{F} \cdot \mathbf{n}\, dS$$

where $\mathbf{n}$ is the outward pointing unit normal vector to the surface S.
Hint : $dS = \sqrt{1 + z_x^2 + z_y^2}$, $\phi = x^2 + y^2 + z^2 = 4$, $\mathbf{n} = \nabla\phi / |\nabla\phi|$.

[9 marks]

Question 5

(a) Use Green's theorem to evaluate

$$\oint_C (3x^2 - 8y^2)\, dx + (4y + 6xy)\, dy$$

where C is the boundary of the region R bounded by $y = \sqrt{x}$ and $y = x^2$ in the counter-clockwise direction.

[6 marks]

(b) Use line integration to evaluate

$$\iint_S \nabla \times \mathbf{F} \cdot \mathbf{n}\, dS$$

where $\mathbb{F} = y\,\mathbf{i}$ and S is the hemisphere given by $x^2 + y^2 + z^2 = 1,\ \ z \geq 0$.
Hint : Stokes' theorem.

[8 marks]

(c) Classify the following functions as either even, odd, or neither.

(i) $1 + x^2 + 3x^4$

(ii) $\sinh(x)$

(iii) $x^2 + \sin(x)$

(iv) $\cosh(x)$

[2 marks]

(d) A periodic function with period 2π is defined within the interval $-\pi < t < \pi$ by

$$f(t) = \begin{cases} -1, & -\pi < t < 0 \\ 1, & 0 \leq t < \pi \end{cases}$$

Express the function as an infinite Fourier series.

[9 marks]

— THE END —