INTI
International College Penang

FINAL
Examination Paper

(COVER PAGE)

Session : **January 2018**

Programme : Diploma in Electrical and Electronic Engineering (DEEI)

Course : **MAT1136: Engineering Mathematics 3**

Date of Examination : 5 March 2018 (Monday)

Time : 11:00am – 1:00pm Reading Time : Nil

Duration : 2 hours

Special Instructions :

This paper consists of **FIVE (5)** questions. Answer any **FOUR (4)** questions in the answer booklet provided. All questions carry equal marks.

Materials permitted :

Non-Programmable Calculator

Materials provided :

Formula Booklet 1

Examiner(s) : **Bark Chee Beng**

Moderator : Dr. Ch'ng Pei Eng

*This paper consists of **5** printed pages, including the cover page.*

INTI INTERNATIONAL COLLEGE PENANG
DIPLOMA IN ELECTRICAL AND ELECTRONIC ENGINEERING (DEEI)
MAT1136 : ENGINEERING MATHEMATICS 3
FINAL EXAMINATION: JANUARY 2018 SESSION

Instructions: This paper consists of **FIVE (5)** questions. Answer any **FOUR (4)** questions in the answer booklet provided. All questions carry equal marks.

Question 1

(a) Given a system of linear equations :

$$x+3y-4z+4w=4$$
$$x+4y-7z+6w=3$$

(i) Use the elementary row operations to determine the rank, and hence the type of solution.

(3 marks)

(ii) Find the solution and leave your answer in vector form.

(4 marks)

(b) Let matrix $\mathbf{A}=\begin{pmatrix}1 & 3\\ 4 & -3\end{pmatrix}$.

(i) Find the eigenvalues of **A**.

(3 marks)

(ii) Find a set of linearly independent eigenvectors for **A**.

(9 marks)

(c) Consider the following system of linear equations :

$$8x-y+2z=21$$
$$2x-11y-z=36$$
$$x-y+9z=14$$

(i) Set up Jacobi scheme for the system.

(3 marks)

(ii) Compute one (1) iteration, starting with initial guess $x^{(0)}=1,\ y^{(0)}=1,\ z^{(0)}=1$.
Keep 3 decimal places in all calculations.

(3 marks)

Question 2

(a) Use Cramer's rule to solve the following system of linear equations :

$$\begin{aligned} 3x - 2y + 3z &= 17 \\ -3x + y - z &= -12 \\ 2x - 2y - z &= 6 \end{aligned}$$

(11 marks)

(b) Given that $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = -4$. Find $\begin{vmatrix} -3c & -3b & -3a \\ 2f & 2e & 2d \\ i-5f & h-5e & g-5d \end{vmatrix}$.

(4 marks)

(c) Consider vector field $\mathbf{F} = (2xy + 2x)\,\mathbf{i} + (x^2 - 6y)\,\mathbf{j}$

(i) Show that **F** is conservative.

(2 marks)

(ii) Find scalar field f such that $\mathbf{F} = \nabla f$.

(6 marks)

(iii) Evaluate the line integral $\int_C \mathbf{F} \bullet d\mathbf{r}$ where C is any path from the point (0, 0) to the point (2, 1).

(2 marks)

Question 3

(a) Given that $\phi = \phi(x, y, z) = x^2 y^3 e^z$.

(i) Find the directional derivative of ϕ at Q(-1, 2, 1) in the direction of the vector $\mathbf{A} = \mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$.

(6 marks)

(ii) In what direction from Q is the directional derivative maximum? Find the magnitude of this maximum value.

(2 marks)

(b) Given that

$\mathbf{A} = xyz\mathbf{i} + (xy + yz)\mathbf{j} + yz^2\mathbf{k}$

$\phi = 2xy^2 - y^2 z$.

Find the following at the point (1, -2, 1):

(i) $\nabla \bullet \mathbf{A}$

(3 marks)

(ii) $\mathbf{A} \bullet \nabla\phi$

(3 marks)

(c) Evaluate $I = \int_P^Q (3x - y + 1)dx - (x + 4y + 2)dy$ along each of the following paths. Hence, justify whether I is independent of path.

(i) a straight line form P(1, 3) to Q(3, 7),

(5 marks)

(ii) the parabola $x = t + 1, y = t^2 + 3, t \in [0,\ 2]$

(6 marks)

Question 4

(a) Use the triple integral $V = \iiint_D dV$ the volume of a solid tetrahedron bounded by closed region D of the planes defined by $x = 0, y = 0, z = 0$ and $x + y + z = 4$.

(5 marks)

(b) Consider the line integral

$$\oint_C xydx + (x^2 - y^2)dy$$

where C is the triangular path OAB defined by O(0, 0), A(1, 0) and B(1, 1) in the counter-clockwise direction. Evaluate the integral

(i) by direct method,

(10 marks)

(ii) by Green's theorem.

(10 marks)

Question 5

(a) Evaluate the line integral $\int \mathbf{F}.d\mathbf{R}$, where $\mathbf{F} = xy^2\mathbf{i} + x^2y\mathbf{j}$, alone the path of the parabolic arc $y = x^2$ connecting the points (0, 0) ad (3, 9).

(7 marks)

(b) Using the Divergence theorem to find $\iint_S \mathbf{F} \bullet d\mathbf{S}$ where

$$\mathbf{F} = y^2z\mathbf{i} + y^3\mathbf{j} + xz\mathbf{k}$$

and S is the boundary of cube defined by $-1 \le x \le 1,\ -1 \le y \le 1,$ and $0 \le z \le 2$.

(6 marks)

(c) Let $f(x)$ be a function of period 2π such that

$$f(x) = \begin{cases} -1, & -\pi < x < 0 \\ 1, & 0 < x < \pi \end{cases}.$$

(i) Sketch a graph of $f(x)$ in the interval $-2\pi < x < 2\pi$.

(2 marks)

(ii) Show that the Fourier series for $f(x)$ in the interval $-\pi < x < \pi$ is

$$f(x) = \frac{4}{\pi}\left[\sin x + \frac{1}{3}\sin 3x + \frac{1}{5}\sin 5x + \ldots\right].$$

(10 marks)

-- THE END --