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FINAL
Examination Paper

(COVER PAGE)

Session	:	**AUGUST 2014**
Programme	:	Diploma in Electrical and Electronic Engineering (DEEI)
Course	:	**MAT 1123: Engineering Mathematics 3**
Date of Examination	:	December 12, 2014 (Friday)
Time	:	11.00am – 1.00pm Reading Time : Nil
Duration	:	2 Hours
Special Instructions	:	

This paper consists of **FIVE (5)** questions. Answer any **FOUR (4)** questions in the answer booklet provided. All questions carry equal marks.

Materials permitted :

Non-programmable calculator

Materials provided :

Formula Booklet 1

Examiner(s)	:	**Chan Ah Wah**
Moderator	:	**Dr. Ch'ng Pei Eng**

This paper consists of 5 printed pages, including the cover page.

INTI INTERNATIONAL COLLEGE PENANG

DIPLOMA IN ELECTRICAL AND ELECTRONIC ENGINEERING (DEEI)

MAT1123 ENGINEERING MATHEMATICS 3

FINAL EXAMINATION : AUGUST 2014 SESSION

Instructions

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Question 1

(a) Evaluate the following determinant .

$$\begin{vmatrix} 1 & 1 & 2 & -3 \\ 3 & 4 & -1 & 5 \\ 4 & 5 & 1 & 2 \\ 1 & 1 & 2 & 1 \end{vmatrix}$$

[7 marks]

(b) Use **rank test** to determine the values of p and q if the following system of equations has **NO** solution.

$$\begin{bmatrix} 1 & 1 & -2 \\ 0 & 1 & p \\ 0 & 2 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ q \end{bmatrix}$$

[6 marks]

(c) Consider the matrix

$$\mathbf{A} = \begin{bmatrix} 5 & 6 & 2 \\ 0 & -1 & -8 \\ 1 & 0 & -2 \end{bmatrix}.$$

Given that $\lambda = 3$ is a **repeated** eigenvalue of **A** .

(i) Find the linearly independent eigenvector(s) corresponding to $\lambda = 3$. Hint: solve the equation $(\mathbf{A} - 3\mathbf{I})\mathbf{V} = \mathbf{0}$.

[6 marks]

(ii) Hence, explain whether **A** is diagonalizable .

[6 marks]

Question 2

(a) Given the system of equations

$$\begin{aligned} b\cos\gamma + c\cos\beta \qquad\qquad &= a \\ a\cos\gamma \qquad\qquad + c\cos\alpha &= b \\ a\cos\beta + b\cos\alpha &= c \end{aligned}$$

where $a, b, c, \alpha, \beta, \gamma$ are real constants with $abc \neq 0$. Use Cramer's rule to show that

$$\cos\alpha = \frac{b^2 + c^2 - a^2}{2bc}.$$

[6 marks]

(b) Determine adj(**A**) if matrix **A** is given by

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & -2 \\ -3 & 1 & 4 \\ 2 & -3 & 4 \end{bmatrix}.$$

Show all your workings .

[6 marks]

(c) Solve the following system of linear equations using Gauss-Seidel method. Compute two (2) iterations, starting with the initial guess $x_1^{(0)} = 1,\ x_2^{(0)} = 1,\ x_3^{(0)} = 1$. Keep 4 decimal places in all calculations .

$$\begin{aligned} 8x_1 - \ \ x_2 + 2x_3 &= 21 \\ 2x_1 - 11x_2 - \ \ x_3 &= 36 \\ x_1 - \ \ x_2 + 9x_3 &= 14 \end{aligned}$$

[7 marks]

(d) A double integral is given by

$$\int_0^1 \int_{x^2}^{2-x} g(x,y)\, dy dx.$$

(i) Sketch and label the region of integration.

[4 marks]

(ii) Hence, rewrite the limits of the integral if the order of integration is reversed.

[2 marks]

Question 3

(a) Given $\mathbf{F} = \langle yz + y,\ xz + x + z,\ xy + y \rangle$.

(i) Show that $\mathbf{F}$ is a conservative vector field .

(ii) Find the scalar potential of $\mathbf{F}$.

[8 marks]

(b) Given that $\mathbf{G} = (x + y)\mathbf{i} - 2y\mathbf{j}$, evaluate the line integral $\int_C \mathbf{G} \bullet d\mathbf{r}$ from $(0,1)$ to $(1,2)$ along the parabola $x = t,\ y = t^2 + 1$ such that $0 \le t \le 1$.

[8 marks]

(c) Use Green's theorem to evaluate

$$\oint_C (2xy - 4y^2)\ dx + (3x - 8xy)\ dy$$

where C is the counter-clockwise oriented boundary of the region R bounded by the curve $y = \sqrt{x}$ and the line $y = x$.

[9 marks]

Question 4

(a) If $\phi = xy^2z^3$ and $\mathbf{A} = \langle xy, -2z^2, xy^2 \rangle$, find each of the following at the point $(1, -2, 1)$.

(i) $\nabla\phi$

(ii) $\nabla \bullet \mathbf{A}$

(iii) $\nabla \times \mathbf{A}$

[11 marks]

(b) Use Stokes' theorem to evaluate

$$\oint_C \mathbf{F} \bullet d\mathbf{r}$$

where $\mathbf{F} = \langle 2y,\ 3x, -z^2 \rangle$ is a vector field acting on the surface of the upper half of the sphere

$$x^2 + y^2 + z^2 = 9$$

and around its boundary C oriented counter-clockwise as viewed from above.

[14 marks]

Question 5

(a) Use the Divergence Theorem to evaluate

$$\iint_S \mathbf{F} \bullet d\mathbf{S}$$

where $\mathbf{F} = \langle x^2,\ 3y^2,\ z^3 \rangle$ and S is the surface of the cube bounded by the planes

$$x = 0, \quad x = 2,$$
$$y = 0, \quad y = 2,$$
$$z = 0, \quad z = 2.$$

[10 marks]

(b) Expand in an infinite Fourier series the following periodic function:

$$f(x) = \begin{cases} 0, & -\pi < x < 0 \\ \pi - x, & 0 \le x < \pi \end{cases}$$

$$f(x) = f(x + 2\pi)$$

[15 marks]

End of Paper

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