INTI International College Penang
LAUREATE INTERNATIONAL UNIVERSITIES

FINAL
Examination Paper

(COVER PAGE)

Session	:	**April 2016**
Programme	:	Diploma in Electrical and Electronic Engineering (DEEI)
Course	:	**MAT 1123: Engineering Mathematics 3**
Date of Examination	:	27 July 2016, Wednesday
Time	:	11.00am – 1.00pm Reading Time : Nil
Duration	:	2 Hours

Special Instructions :

This paper consists of **FIVE (5)** questions. Answer any **FOUR (4)** questions in the answer booklet provided. All questions carry equal marks.

Materials permitted :

Non-programmable calculator

Materials provided :

Formula Booklet 1

Examiner(s)	:	**Mr. Chan Ah Wah**
Moderator	:	Dr. Ch'ng Pei Eng

This paper consists of **5** printed pages, including the cover page.

INTI INTERNATIONAL COLLEGE PENANG
DIPLOMA IN ELECTRICAL AND ELECTRONIC ENGINEERING (DEEI)
MAT1123 ENGINEERING MATHEMATICS 3
FINAL EXAMINATION : APRIL 2016 SESSION

Instructions

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Question 1

(a) Given the following system of linear equations :

$$\begin{aligned} x + 2y + 3z &= 0 \\ x + 3y + 5z &= 1 \\ 2x + 3y + 4z &= -1 \end{aligned}$$

(i) Determine the reduced row echelon form (rref) of the system .

(ii) Use Rank Test to prove that it is consistent with infinitely many solutions .

(iii) Solve the system and express the solution in vector form .

[10 marks]

(b) Given a homogeneous system as follows :

$$\begin{aligned} x + 2y + kz &= 0 \\ y + 2z &= 0 \\ x \qquad - z &= 0 \end{aligned}$$

Find the value of k if the homogeneous system has non-trivial solutions .

Hint: Use determinant approach .

[6 marks]

(c) Given a 2×2 matrix $\mathbf{A} = \begin{bmatrix} 5 & 4 \\ 3 & 1 \end{bmatrix}$.

(i) Find the eigenvalues of $\mathbf{A}$.

[3 marks]

(ii) Find an eigenvector corresponding to each of the eigenvalues .

[6 marks]

Question 2

(a) Let $\mathbf{A}$ be a 2×2 matrix. If $\det(\mathbf{A}) = 5$, find $\det(3\mathbf{A}^{-1})$ where $\mathbf{A}^{-1}$ is the inverse of $\mathbf{A}$.

[4 marks]

(b) Find $\mathbf{A}^{-1}$ using elementary row operations if the matrix $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{bmatrix} 1 & 3 & -1 \\ -1 & 0 & 0 \\ 0 & 2 & -1 \end{bmatrix}$$

[8 marks]

(c) Determine $\operatorname{adj}(\mathbf{A})$ if matrix $\mathbf{A}$ is given by

$$A = \begin{bmatrix} 1 & 0 & -2 \\ -3 & 1 & 4 \\ 2 & -3 & 4 \end{bmatrix}.$$

Show all your workings.

[6 marks]

(d) Rearrange the equations (if necessary) and apply Gauss-Seidel scheme to solve the following system. Complete three (3) iterations starting with the initial guess of $x_1^{(0)} = 1$, $x_2^{(0)} = 1$, $x_3^{(0)} = 1$. Maintain 4 decimal places throughout the calculations.

$$\begin{aligned} -x_1 + x_2 + 8x_3 &= 33 \\ 10x_1 - x_2 + x_3 &= 21 \\ x_1 + 9x_2 - x_3 &= 25 \end{aligned}$$

[7 marks]

Question 3

(a) Let $\mathbf{A} = \mathbf{i} + 2\mathbf{j} + \mathbf{k}$ and $\mathbf{B} = 2\mathbf{i} - \mathbf{j} - \mathbf{k}$. Find a vector of magnitude 5 that is perpendicular to both $\mathbf{A}$ and $\mathbf{B}$.

[5 marks]

(b) Let $\mathbf{A} = xy^3z\mathbf{i} - x^2y\mathbf{j} + zy^3\mathbf{k}$ be a vector field. Find $\nabla(\nabla \cdot \mathbf{A})$ at the point $(1,1,1)$.

[5 marks]

(c) Given a scalar field $\phi(x,y,z) = xy^2z + x^2z^3$. Find the directional derivative of ϕ at the point $(1,1,1)$ in the direction of the vector $\mathbf{A} = \mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$.

[6 marks]

(d) Use Gauss' Divergence Theorem to evaluate $\iint_S \mathbf{F} \cdot d\mathbf{S}$, where

$$\mathbf{F} = 3xyz\mathbf{i} + 4y^2z\mathbf{j} + 2yz^2\mathbf{k}$$

and S is the closed surface of a cuboid enclosed by the planes $x = 0,\ x = 3,\ y = 0,\ y = 2,$ $z = 0,\ z = 1$.

[9 marks]

Question 4

(a) A force $\mathbf{F} = 2z\mathbf{i} + 5x\mathbf{j} - 4y\mathbf{k}$ displaces a particle in space from $(0,0,0)$ to $(2,1,3)$ along the curve C : $x = 2t,\ \ y = t^2,\ \ z = 3t$. Find the work done by the force given by the integral $\int_C \mathbf{F} \cdot d\mathbf{r}$.

[7 marks]

(b) Given $\mathbf{F} = (yz + y)\mathbf{i} + (xz + x + z)\mathbf{j} + (xy + y)\mathbf{k}$.

(i) Show that **F** is a conservative vector field .

[4 marks]

(ii) Find the scalar potential of **F** .

[5 marks]

(c) Let $S : x^2 + y^2 + z^2 = 4$ be a hemisphere above the xy-plane. Suppose a force $\mathbf{F} = 2\mathbf{k}$ acts on the surface and around its boundary. Evaluate the double integral

$$\iint_S \mathbf{F} \cdot \mathbf{n}\, dS$$

where **n** is the outward pointing unit normal vector to the surface S.
Hint : $dS = \sqrt{1 + z_x^2 + z_y^2},\ \ \phi = x^2 + y^2 + z^2 = 4,\ \ \mathbf{n} = \nabla\phi / |\nabla\phi|$.

[9 marks]

Question 5

(a) Use Green's theorem to evaluate

$$\oint_C (3x^2 - 8y^2)\, dx + (4y + 6xy)\, dy$$

where C is the boundary of the region R bounded by $y = \sqrt{x}$ and $y = x^2$ in the counter-clockwise direction .

[6 marks]

(b) Use line integration to evaluate

$$\iint_S \nabla \times \mathbf{F} \cdot \mathbf{n}\, dS$$

where $\mathbf{F} = y\,\mathbf{i}$ and S is the hemisphere given by $x^2 + y^2 + z^2 = 1,\ \ z \geq 0$.
Hint : Stokes' theorem .

[8 marks]

(c) Classify the following functions as either even, odd, or neither .

(i) $1 + x^2 + 3x^4$

(ii) $\sinh(x)$

(iii) $x^2 + \sin(x)$

(iv) $\cosh(x)$

[2 marks]

(d) A periodic function with period 2π is defined within the interval $-\pi < t < \pi$ by

$$f(t) = \begin{cases} -1, & -\pi < t < 0 \\ 1, & 0 \leq t < \pi \end{cases}$$

Express the function as an infinite Fourier series .

[9 marks]

End of Paper

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