INTI International College Penang
LAUREATE INTERNATIONAL UNIVERSITIES*

FINAL
Examination Paper

(COVER PAGE)

Session	:	**April 2016**
Programme	:	Diploma In Electrical And Electronic Engineering (DEEI)
Course	:	**EEE2104: Electromagnetic Field Theory**
Date of Examination	:	27 July 2016, Wednesday
Time	:	2.00pm – 4.00pm
Duration	:	2 Hours Reading Time : Nil
Special Instructions	:	

This paper consists of **SIX (6)** questions. Answer any **FOUR (4)** questions in the answer booklet provided. All questions carry equal marks.

IMPORTANT NOTE : **THIS PAPER SHOULD NOT BE TAKEN OUT OF THE EXAMINATION HALL**

Materials Permitted :
Non Programmable Scientific Calculator

Materials Provided :
Appendix

Examiner(s)	:	**Ms. Koh Yeok Ping**
Moderator	:	Dr. Ooi Beng Lee

This paper consists of 5 printed pages, including the cover page.

INTI INTERNATIONAL COLLEGE

DIPLOMA IN ELECTRICAL AND ELECTRONIC ENGINEERING (DEEI)
EEE2104: ELECTROMAGNETIC FIELD THEORY
FINAL EXAMINATIONS: APRIL 2016 SESSION

Instructions: This paper consists of **SIX (6)** questions. Answer any **FOUR (4)** questions in the answer booklet provided. All questions carry equal marks. The marks allocated to each sub-question are shown in the brackets at the right-hand margin.

Question 1

a. A thin annular disc of inner radius a and outer radius b carries a uniform surface charge density $\rho_s\ C/m^2$ and is placed on the xy-plane with axis same as the z-axis as shown in Figure Q1(a) below.

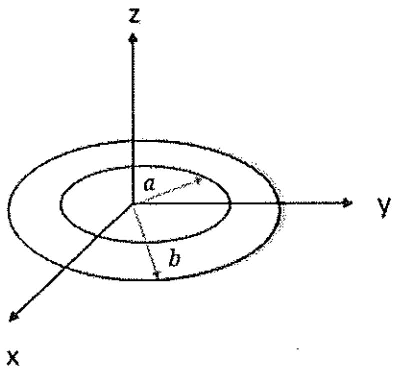

Figure Q1(a)

i. Show that

$$\bar{E}(0,0,h) = \frac{\rho_s h}{2\varepsilon_0}\left[\frac{1}{\sqrt{a^2+h^2}} - \frac{1}{\sqrt{b^2+h^2}}\right]\bar{a}_z\ N/C$$ (9 marks)

ii. Find $\bar{E}$ by letting $a \to 0, b \to \infty$. (3 marks)

b. Charge is distributed uniformly along an infinite straight line with constant density $\rho_L\ C/m$ along the $z-$axis. Using Gauss' Law, develop the expression for $\bar{D}$ and $\bar{E}$ at general point P. (6 marks)

c. Two standard charge distributions as below:
- A uniform sheet at $x = 0$ with $\rho_s = \frac{1}{3\pi} n\ C/m^2$
- A uniform line at $x = 6, y = 0$ with $\rho_L = -2nC/m$

Determine $\bar{E}$ and $\bar{D}$ at (2, 0, 2) due to both charge distributions. (7 marks)

Question 2

a. Define electric potential between two points. (2 marks)

b. Find the potential at point (0,0,5) with respect to point (0,0,15) that due to a point charge 500pC at the origin. (5 marks)

c. Find the work done in moving a point charge $Q = 5uC$ from the origin to $(2, \pi/4, \pi/2)$ in spherical coordinates, in the field $\bar{E} = 5e^{-r/4}\bar{a}_r + \frac{10}{r\sin\theta}\bar{a}_\phi$ (V/m). (10 marks)

d. Given that $\bar{E}_1 = 5\bar{a}_x - 2\bar{a}_y + 3\bar{a}_z \; kV/m$ exists for $z \geq 0$. See Figure Q2(d) below. Find $\bar{E}_2$ and the angles θ_1 and θ_2. (8 marks)

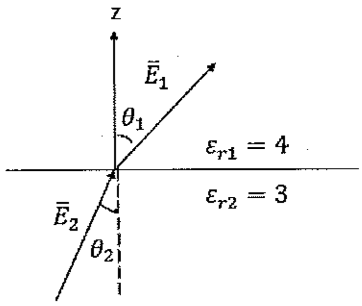

Figure Q2(d)

Question 3

a. A parallel-plate capacitor with area $0.30m^2$ and separation 5.5 mm contains three dielectrics with interfaces normal to $\bar{E}$ and $\bar{D}$ as follows:
$\varepsilon_{r1} = 3.0$, $d_1 = 1.0mm$, $\varepsilon_{r2} = 4.0$, $d_2 = 2.0mm$, $\varepsilon_{r3} = 6.0$, $d_3 = 2.5mm$
Determine the total capacitance. (9 marks)

b. Consider length L of two coaxial conductors of inner radius a and outer radius b filled with a homogeneous dielectric with permittivity ε as shown in Figure Q3(b). Prove that the capacitance is equal to: (12 marks)

$$C = \frac{2\pi\varepsilon L}{\ln\frac{b}{a}}$$

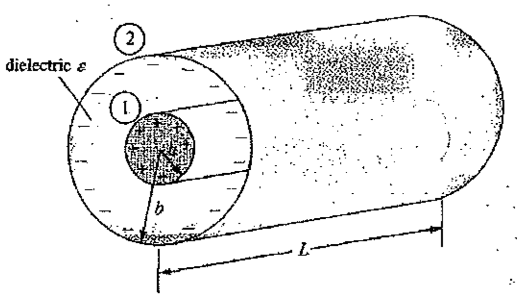

Figure Q3(b)

c. Find the capacitance per unit length of a coaxial cable with outer radius of 4mm and inner radius of 0.5mm if the dielectric has $\varepsilon_r = 5.2$. (4 marks)

Question 4

a. State Biot-Savart's Law. (2 marks)

b. An infinite current line is along z-axis and carries current I_o. By using Biot-Savart's Law, show that $\bar{H}$ at any point of $(\rho, \emptyset, z)$ is (6 marks)

$$\bar{H} = \frac{I_0}{2\pi\rho}\bar{a}_\emptyset$$

c. A circular conductor of radius $\rho = 1cm$ has an internal field

$$\bar{H} = \frac{10^4}{\rho}\left(\frac{1}{a^2}\sin a\rho - \frac{\rho}{a}\cos a\rho\right)\bar{a}_\emptyset \quad A/m$$

where $a = \frac{\pi}{2\rho}$. Find the total current in the conductor. (5 marks)

d. A radical field

$$\bar{H} = \frac{2.39\times10^6}{\rho}\cos\emptyset\bar{a}_\rho \ A/m$$

exists in free space. Find the magnetic flux φ crossing the surface defined by $-\pi/4 \leq \emptyset \leq \pi/4$ and $0 \leq z \leq 1$. (6 marks)

e. A current distribution gives rise to the vector magnetic potential

$$\bar{A} = x^2y\bar{a}_x + y^2x\bar{a}_y - 4xyz\bar{a}_z \ Wb/m$$

Calculate the magnetic flux φ through the surface defined by $z = 1,\ 0 \leq x \leq 1$ and $-1 \leq y \leq 4$. (6 marks)

Question 5

a. An electromagnet of cross section $4cm^2$ has a tightly wound coil with 1500 turns. The inner and the outer radius of the magnetic core are 10cm and 12cm respectively. The length of the air gap is 1cm as shown in Figure Q5(a). If the current in the coil is 4A and the relative permeability of the magnetic material is 1200, determine the flux density in the magnetic circuit. Given that air relative permeability is 1. (9 marks)

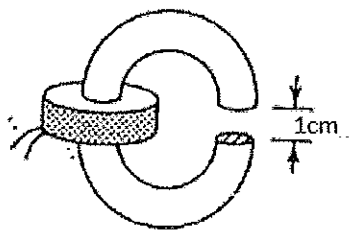

Figure Q5(a)

b. Explain how to classify magnetic material according to the magnetic susceptibility and characteristic. (9 marks)

c. A very long solenoid with $4cm^2$ cross section has an iron core($\mu_r = 1000$) and 4000 turns/meter. If it carries a current of 500mA, find:
   i. Its self inductance per meter (4 marks)
   ii. The energy per meter stored in its field (3 marks)

Question 6

a. State Faraday's Law. (2 marks)

b. In a material for which $\sigma = 5.0\ S/m$ and $\varepsilon_r = 1$, the electric field intensity is given as

$$E = 250 \sin 10^{10}t\ V/m$$

Find:
   i. The conduction current density (2 marks)
   ii. The displacement current density (4 marks)
   iii. The frequency at which they have equal magnitudes (4 marks)

c. The loop shown in Figure Q6(c) is inside a uniform magnetic field $\bar{B} = 50\bar{a}_x\ Wb/m^2$. If side *DC* of the loop cuts the flux lines at the frequency of 50Hz and the loop lies in the *yz*-plane at time $t = 0$, find the induced emf at $t = 1$ms. (8 marks)

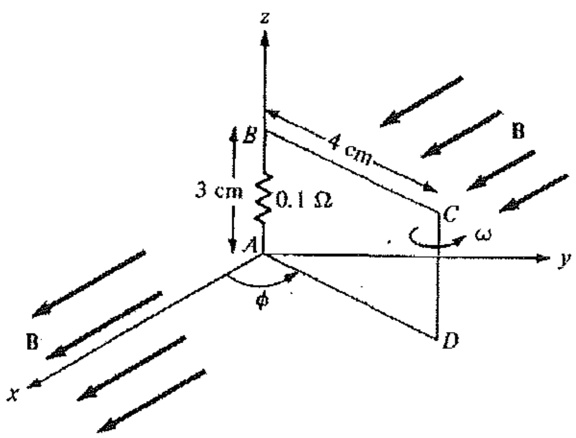

Figure Q6(c)

d. Given $\bar{E} = E_m \sin(\omega t - \beta z)\bar{a}_y$, in free space, find $\bar{H}$. (5 marks)

~ The End ~

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MATHEMATICAL FORMULAS AND PHYSICAL CONSTANT

Appendix A: Physical Constant

Permittivity of free space, $\varepsilon_o = 8.854 \times 10^{-12}$ F/m $\cong \dfrac{10^{-9}}{36\pi}$ F/m

Permeability of free space, $\mu_o = 12.6 \times 10^{-7}$ H/m $\cong 4\pi \times 10^{-7}$ H/m

Appendix B: Trigonometry Identities

Sum or difference of two angles:

$$\sin(a \pm b) = \sin a \cos b \pm \cos a \sin b$$

$$\cos(a \pm b) = \cos a \cos b \mp \sin a \sin b$$

$$\tan(a \pm b) = \frac{\tan a \pm \tan b}{1 \mp \tan a \tan b}$$

Double angle formulas:

$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$

$$\sin 2\theta = 2\sin\theta\cos\theta \qquad \cos 2\theta = 2\cos^2\theta - 1$$

$$\cos 2\theta = 1 - 2\sin^2\theta \qquad \cos 2\theta = \cos^2\theta - \sin^2\theta$$

Pythagorean Identities:

$$\sin^2\theta + \cos^2\theta = 1$$

$$\tan^2\theta + 1 = \sec^2\theta \qquad \cot^2\theta + 1 = \csc^2\theta$$

Half angle formulas:

$$\sin^2\theta = \frac{1}{2}(1-\cos 2\theta) \qquad \cos^2\theta = \frac{1}{2}(1+\cos 2\theta)$$

$$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}} \qquad \cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}$$

$$\tan\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}} = \frac{\sin\theta}{1+\cos\theta} = \frac{1-\cos\theta}{\sin\theta}$$

Appendix C: Differential Length, Area and Volume

Cartesian System :

Differential displacement, $d\bar{l} = dx\bar{a}_x + dy\bar{a}_y + dz\bar{a}_z$

Differential normal area,

$$d\bar{s}_x = dydz\bar{a}_x$$
$$d\bar{s}_y = dxdz\bar{a}_y$$
$$d\bar{s}_z = dxdy\bar{a}_z$$

Differential volume, $dv = dxdydz$

Cylindrical System :

Differential displacement, $d\bar{l} = d\rho\bar{a}_\rho + \rho d\phi\bar{a}_\phi + dz\bar{a}_z$

Differential normal area, $d\bar{S} = \rho\, d\phi\, dz\, \bar{a}_\rho$
$$= d\rho\, dz\, \bar{a}_\phi$$
$$= \rho\, d\phi\, d\rho\, \bar{a}_z$$

Differential volume, $dv = \rho d\rho d\phi dz$

Spherical System :

Differential displacement, $d\bar{l} = dr\bar{a}_r + rd\theta\bar{a}_\theta + r\sin\theta\, d\phi\, \bar{a}_\phi$

Differential normal area, $d\bar{S} = r^2 \sin\theta\, d\theta\, d\phi\, \bar{a}_r$
$$= r\sin\theta\, dr\, d\phi\, \bar{a}_\theta$$
$$= r\, dr\, d\theta\, \bar{a}_\phi$$

Differential volume, $dv = r^2 \sin\theta\, drd\theta d\emptyset$

Appendix D: Gradient of scalar

Cartesian System : $\nabla V = \frac{\partial V}{\partial x}\bar{a}_x + \frac{\partial V}{\partial y}\bar{a}_y + \frac{\partial V}{\partial z}\bar{a}_z$

Cylindrical System : $\nabla V = \frac{\partial V}{\partial \rho}\bar{a}_\rho + \frac{1}{\rho}\frac{\partial V}{\partial \phi}\bar{a}_\phi + \frac{\partial V}{\partial z}\bar{a}_z$

Spherical System : $\nabla V = \frac{\partial V}{\partial r}\bar{a}_r + \frac{1}{r}\frac{\partial V}{\partial \theta}\bar{a}_\theta + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}\bar{a}_\phi$

Appendix E: Curl of a Vector

Cartesian System : $\nabla \times \bar{A} = \begin{vmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$

Cylindrical System : $\nabla \times \bar{A} = \frac{1}{\rho}\begin{vmatrix} \bar{a}_\rho & \rho\bar{a}_\phi & \bar{a}_z \\ \frac{\partial}{\partial \rho} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ A_\rho & \rho A_\phi & A_z \end{vmatrix}$

Spherical System : $\nabla \times \bar{A} = \frac{1}{r^2\sin\theta}\begin{vmatrix} \bar{a}_r & r\bar{a}_\theta & r\sin\theta\bar{a}_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ A_r & rA_\theta & r\sin\theta\, A_\phi \end{vmatrix}$

Appendix F: Divergence of a vector

Cartesian System : $\nabla \cdot \bar{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$

Cylindrical System : $\nabla \cdot \bar{A} = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho A_\rho) + \frac{1}{\rho}\frac{\partial}{\partial \phi}(A_\phi) + \frac{\partial}{\partial z}(A_z)$

Spherical System : $\nabla \cdot \bar{A} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(A_\theta \sin\theta) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}(A_\phi)$

Appendix G: Laplacian of a vector

Cartesian System:

$$\nabla^2 \overline{A} = \left(\frac{\partial^2 A_x}{\partial x^2} + \frac{\partial^2 A_x}{\partial y^2} + \frac{\partial^2 A_x}{\partial z^2} \right) \overline{a}_x + \left(\frac{\partial^2 A_y}{\partial x^2} + \frac{\partial^2 A_y}{\partial y^2} + \frac{\partial^2 A_y}{\partial z^2} \right) \overline{a}_y + \left(\frac{\partial^2 A_z}{\partial x^2} + \frac{\partial^2 A_z}{\partial y^2} + \frac{\partial^2 A_z}{\partial z^2} \right) \overline{a}_z$$

Appendix H: Relationship between Cartesian and Cylindrical System

$$\rho = \sqrt{x^2 + y^2} \ , \quad \tan\phi = \frac{y}{x} \quad , \ z = z$$

or

$$x = \rho\cos\phi \ , \ y = \rho\sin\phi \ , \ z = z$$

$$\overline{a}_x = \cos\phi\,\overline{a}_\rho - \sin\phi\,\overline{a}_\phi \qquad \overline{a}_\rho = \cos\phi\,\overline{a}_x + \sin\phi\,\overline{a}_y$$

$$\overline{a}_y = \sin\phi\,\overline{a}_\rho + \cos\phi\,\overline{a}_\phi \qquad \overline{a}_\phi = -\sin\phi\,\overline{a}_x + \cos\phi\,\overline{a}_y$$

$$\overline{a}_z = \overline{a}_z \qquad \overline{a}_z = \overline{a}_z$$

Appendix I: Vector relationship between Cartesian and Spherical System

$$r = \sqrt{x^2 + y^2 + z^2} \ , \quad \tan\theta = \frac{\sqrt{x^2 + y^2}}{z} \quad , \ \tan\phi = \frac{y}{x}$$

or

$$x = r\sin\theta\cos\phi \ , \ y = r\sin\theta\sin\phi \ , \ z = r\cos\theta$$

$$\overline{a}_x = \sin\theta\cos\phi\,\overline{a}_r + \cos\theta\cos\phi\,\overline{a}_\theta - \sin\phi\,\overline{a}_\phi \qquad \overline{a}_r = \sin\theta\cos\phi\,\overline{a}_x + \sin\theta\sin\phi\,\overline{a}_y + \cos\theta\,\overline{a}_z$$

$$\overline{a}_y = \sin\theta\sin\phi\,\overline{a}_r + \cos\theta\sin\phi\,\overline{a}_\theta + \cos\phi\,\overline{a}_\phi \qquad \overline{a}_\theta = \cos\theta\cos\phi\,\overline{a}_x + \cos\theta\sin\phi\,\overline{a}_y - \sin\theta\,\overline{a}_z$$

$$\overline{a}_z = \cos\theta\,\overline{a}_r - \sin\theta\,\overline{a}_\theta \qquad \overline{a}_\phi = -\sin\phi\,\overline{a}_x + \cos\phi\,\overline{a}_y$$

Appendix J: Derivatives of Trigonometric Functions

$$\frac{d}{dx}\sin(x)=\cos(x) \qquad \frac{d}{dx}\cos(x)=-\sin(x)$$

$$\frac{d}{dx}\tan(x)=\sec^2(x) \qquad \frac{d}{dx}\cot(x)=-\csc^2(x)$$

$$\frac{d}{dx}\sec(x)=\sec(x)\tan(x) \qquad \frac{d}{dx}\csc(x)=-\csc(x)\cot(x)$$

Appendix K: Table of Integrals

$$\int\frac{1}{(a^2+u^2)^{3/2}}du=\frac{u}{a^2\sqrt{a^2+u^2}}+C$$

$$\int\frac{u}{(a^2+u^2)^{3/2}}du=-\frac{1}{\sqrt{a^2+u^2}}+C$$

$$\int\frac{1}{a^2+u^2}du=\frac{1}{a}\tan^{-1}\frac{u}{a}$$

$$\int\frac{1}{\sqrt{a^2+u^2}}du=\ln\left(u+\sqrt{a^2+u^2}\right)+C$$

Appendix L: Vector Identities

Gradient

1. $\vec{\nabla}(f+g) = \vec{\nabla} f + \vec{\nabla} g$
2. $\vec{\nabla}(cf) = c\vec{\nabla} f$, for any constant c
3. $\vec{\nabla}(fg) = f\vec{\nabla} g + g\vec{\nabla} f$
4. $\vec{\nabla}(f/g) = \big(g\vec{\nabla} f - f\vec{\nabla} g\big)/g^2$ at points $\vec{\mathbf{x}}$ where $g(\vec{\mathbf{x}}) \neq 0$.
5. $\vec{\nabla}(\vec{\mathbf{F}} \cdot \vec{\mathbf{G}}) = \vec{\mathbf{F}} \times (\vec{\nabla} \times \vec{\mathbf{G}}) - (\vec{\nabla} \times \vec{\mathbf{F}}) \times \vec{\mathbf{G}} + (\vec{\mathbf{G}} \cdot \vec{\nabla})\vec{\mathbf{F}} + (\vec{\mathbf{F}} \cdot \vec{\nabla})\vec{\mathbf{G}}$

Divergence

6. $\vec{\nabla} \cdot (\vec{\mathbf{F}} + \vec{\mathbf{G}}) = \vec{\nabla} \cdot \vec{\mathbf{F}} + \vec{\nabla} \cdot \vec{\mathbf{G}}$
7. $\vec{\nabla} \cdot (c\vec{\mathbf{F}}) = c\vec{\nabla} \cdot \vec{\mathbf{F}}$, for any constant c
8. $\vec{\nabla} \cdot (f\vec{\mathbf{F}}) = f\vec{\nabla} \cdot \vec{\mathbf{F}} + \vec{\mathbf{F}} \cdot \vec{\nabla} f$
9. $\vec{\nabla} \cdot (\vec{\mathbf{F}} \times \vec{\mathbf{G}}) = \vec{\mathbf{G}} \cdot (\vec{\nabla} \times \vec{\mathbf{F}}) - \vec{\mathbf{F}} \cdot (\vec{\nabla} \times \vec{\mathbf{G}})$

Curl

10. $\vec{\nabla} \times (\vec{\mathbf{F}} + \vec{\mathbf{G}}) = \vec{\nabla} \times \vec{\mathbf{F}} + \vec{\nabla} \times \vec{\mathbf{G}}$
11. $\vec{\nabla} \times (c\vec{\mathbf{F}}) = c\vec{\nabla} \times \vec{\mathbf{F}}$, for any constant c
12. $\vec{\nabla} \times (f\vec{\mathbf{F}}) = f\vec{\nabla} \times \vec{\mathbf{F}} + \vec{\nabla} f \times \vec{\mathbf{F}}$
13. $\vec{\nabla} \times (\vec{\mathbf{F}} \times \vec{\mathbf{G}}) = \vec{\mathbf{F}}(\vec{\nabla} \cdot \vec{\mathbf{G}}) - (\vec{\nabla} \cdot \vec{\mathbf{F}})\vec{\mathbf{G}} + (\vec{\mathbf{G}} \cdot \vec{\nabla})\vec{\mathbf{F}} - (\vec{\mathbf{F}} \cdot \vec{\nabla})\vec{\mathbf{G}}$

Laplacian

14. $\vec{\nabla}^2(f+g) = \vec{\nabla}^2 f + \vec{\nabla}^2 g$
15. $\vec{\nabla}^2(cf) = c\vec{\nabla}^2 f$, for any constant c
16. $\vec{\nabla}^2(fg) = f\vec{\nabla}^2 g + 2\vec{\nabla} f \cdot \vec{\nabla} g + g\vec{\nabla}^2 f$

Degree Two

17. $\vec{\nabla} \cdot (\vec{\nabla} \times \vec{\mathbf{F}}) = 0$
18. $\vec{\nabla} \times (\vec{\nabla} f) = 0$
19. $\vec{\nabla} \cdot (\vec{\nabla} f \times \nabla g) = 0$
20. $\vec{\nabla} \cdot (f\vec{\nabla} g - g\vec{\nabla} f) = f\vec{\nabla}^2 g - g\vec{\nabla}^2 f$
21. $\vec{\nabla} \times (\vec{\nabla} \times \vec{\mathbf{F}}) = \vec{\nabla}(\vec{\nabla} \cdot \vec{\mathbf{F}}) - \vec{\nabla}^2\vec{\mathbf{F}}$