



INTI
International College Penang
LAUREATE INTERNATIONAL UNIVERSITIES*

FINAL
Examination Paper

(COVER PAGE)

Session : August 2017

Programme : Diploma in Electrical and Electronic Engineering (DEEI)

Course : **MAT 1136: Engineering Mathematics 3**

Date of Examination : 8 December 2017 (Friday)

Time : 11:00am – 1:00pm Reading Time : Nil

Duration : 2 Hours

Special Instructions :

This paper consists of **FIVE (5)** questions. Answer any **FOUR (4)** questions in the answer booklet provided. All questions carry equal marks.

Materials permitted :

Non-programmable calculator

Materials provided :

Formula Booklet 1

Examiner(s) : **Chan Ah Wah**

Moderator : Dr. Ch'ng Pei Eng

This paper consists of 5 printed pages, including the cover page.

INTI INTERNATIONAL COLLEGE PENANG
 DIPLOMA IN ELECTRICAL AND ELECTRONIC ENGINEERING (DEEI)
 MAT1136: ENGINEERING MATHEMATICS 3
 FINAL EXAMINATION : AUGUST 2017 SESSION

Instructions

This paper consists of FIVE (5) questions. Answer any FOUR (4) questions in the answer booklet provided. All questions carry equal marks.

Question 1

(a) Given the following system of linear equations :

$$x + 2y + 3z = 0$$

$$x + 3y + 5z = 1$$

$$2x + 3y + 4z = -1$$

- (i) Determine the reduced row echelon form (rref) of the system .
- (ii) Use Rank Test to prove that it is consistent with infinitely many solutions.
- (iii) Solve the system and express the solution in vector form.

[10 marks]

(b) Given a homogeneous system as follows:

$$x + 2y + kz = 0$$

$$y + 2z = 0$$

$$x - z = 0$$

Find the value of k if the homogeneous system has non-trivial solutions.

Hint: Use determinant approach.

[6 marks]

(c) Given a 2×2 matrix $\mathbf{A} = \begin{bmatrix} 5 & 4 \\ 3 & 1 \end{bmatrix}$.

(i) Find the eigenvalues of \mathbf{A} .

[3 marks]

(ii) Find an eigenvector corresponding to each of the eigenvalues.

[6 marks]

Question 2

- (a) Let \mathbf{A} be a 2×2 matrix. If $\det(\mathbf{A}) = 5$, find $\det(3\mathbf{A}^{-1})$ where \mathbf{A}^{-1} is the inverse of \mathbf{A} .

[4 marks]

- (b) Find \mathbf{A}^{-1} using elementary row operations if the matrix \mathbf{A} is given by

$$\mathbf{A} = \begin{bmatrix} 1 & 3 & -1 \\ -1 & 0 & 0 \\ 0 & 2 & -1 \end{bmatrix}.$$

[8 marks]

- (c) Determine $\text{adj}(\mathbf{A})$ if matrix \mathbf{A} is given by

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & -2 \\ -3 & 1 & 4 \\ 2 & -3 & 4 \end{bmatrix}.$$

Show all your workings.

[6 marks]

- (d) Rearrange the equations (if necessary) and apply Gauss-Seidel scheme to solve the following system. Complete three (3) iterations starting with the initial guess of $x_1^{(0)} = 1$, $x_2^{(0)} = 1$, $x_3^{(0)} = 1$. Maintain 4 decimal places throughout the calculations.

$$-x_1 + x_2 + 8x_3 = 33$$

$$10x_1 - x_2 + x_3 = 21$$

$$x_1 + 9x_2 - x_3 = 25$$

[7 marks]

Question 3

- (a) Let $\mathbf{A} = \mathbf{i} + 2\mathbf{j} + \mathbf{k}$ and $\mathbf{B} = 2\mathbf{i} - \mathbf{j} - \mathbf{k}$. Find a vector of magnitude 5 that is perpendicular to both \mathbf{A} and \mathbf{B} .

[5 marks]

- (b) Let $\mathbf{A} = xy^3z\mathbf{i} - x^2y\mathbf{j} + zy^3\mathbf{k}$ be a vector field. Find $\nabla(\nabla \cdot \mathbf{A})$ at the point $(1, 1, 1)$.

[5 marks]

- (c) Given a scalar field $\phi(x, y, z) = xy^2z + x^2z^3$. Find the directional derivative of ϕ at the point $(1, 1, 1)$ in the direction of the vector $\mathbf{A} = \mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$.

[6 marks]

(d) Use Gauss' Divergence Theorem to evaluate $\iint_S \mathbf{F} \cdot d\mathbf{S}$, where

$$\mathbf{F} = 3xyz\mathbf{i} + 4y^2z\mathbf{j} + 2yz^2\mathbf{k}$$

and S is the closed surface of a cuboid enclosed by the planes $x = 0$, $x = 3$, $y = 0$, $y = 2$, $z = 0$, $z = 1$.

[9 marks]

Question 4

(a) A force $\mathbf{F} = 2z\mathbf{i} + 5x\mathbf{j} - 4y\mathbf{k}$ displaces a particle in space from $(0,0,0)$ to $(2,1,3)$ along the curve $C : x = 2t$, $y = t^2$, $z = 3t$. Find the work done by the force given by the integral

$$\int_C \mathbf{F} \cdot d\mathbf{r}.$$

[7 marks]

(b) Given $\mathbf{F} = (yz + y)\mathbf{i} + (xz + x + z)\mathbf{j} + (xy + y)\mathbf{k}$.

(i) Show that \mathbf{F} is a conservative vector field.

[4 marks]

(ii) Find the scalar potential of \mathbf{F} .

[5 marks]

(c) Let $S : x^2 + y^2 + z^2 = 4$ be a hemisphere above the xy -plane. Suppose a force $\mathbf{F} = 2\mathbf{k}$ acts on the surface and around its boundary. Evaluate the double integral

$$\iint_S \mathbf{F} \cdot \mathbf{n} dS$$

where \mathbf{n} is the outward pointing unit normal vector to the surface S .

Hint: $dS = \sqrt{1 + z_x^2 + z_y^2}$, $\phi = x^2 + y^2 + z^2 = 4$, $\mathbf{n} = \nabla\phi/|\nabla\phi|$.

[9 marks]

Question 5

(a) Use Green's theorem to evaluate

$$\oint_C (3x^2 - 8y^2) dx + (4y + 6xy) dy$$

where C is the boundary of the region R bounded by $y = \sqrt{x}$ and $y = x^2$ in the counter-clockwise direction.

[6 marks]

(b) Use line integration to evaluate

$$\iint_S \nabla \times \mathbf{F} \cdot \mathbf{n} \, dS$$

where $\mathbf{F} = y \mathbf{i}$ and S is the hemisphere given by $x^2 + y^2 + z^2 = 1$, $z \geq 0$.

Hint : Stokes' theorem .

[8 marks]

(c) Classify the following functions as either even, odd, or neither.

(i) $1 + x^2 + 3x^4$

(ii) $\sinh(x)$

(iii) $x^2 + \sin(x)$

(iv) $\cosh(x)$

[2 marks]

(d) A periodic function with period 2π is defined within the interval $-\pi < t < \pi$ by

$$f(t) = \begin{cases} -1, & -\pi < t < 0 \\ 1, & 0 \leq t < \pi. \end{cases}$$

Express the function as an infinite Fourier series.

[9 marks]

————— End of Paper —————

<mat1136(F)/aug2017/chanaw>

