



FINAL
Examination Paper

(COVER PAGE)

Session : August 2016

Programme : Diploma in Electrical and Electronic Engineering (DEEI)

Course : MAT 1123: Engineering Mathematics 3

Date of Examination : 13 December 2016 (Tuesday)

Time : 11:00am – 1:00pm Reading Time : Nil

Duration : 2 Hours

Special Instructions :

This paper consists of FIVE (5) questions. Answer any FOUR (4) questions in the answer booklet provided. All questions carry equal marks.

Materials permitted :

Non-programmable calculator

Materials provided :

Formula Booklet 1

Examiner(s) :

Chan Ah Wah

Moderator :

Dr. Ch'ng Pei Eng

This paper consists of 6 printed pages, including the cover page.

INTI INTERNATIONAL COLLEGE PENANG
 DIPLOMA IN ELECTRICAL AND ELECTRONIC ENGINEERING (DEEI)
 MAT1123 ENGINEERING MATHEMATICS 3
 FINAL EXAMINATION : AUGUST 2016 SESSION

Instructions

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Question 1

- (a) Consider the following system of linear equations where β is a real constant.

$$\begin{aligned}x - 3y + 2z &= 2 \\ -2x + 7y &= \beta^2 \\ 3x - 10y + \beta z &= -2\end{aligned}$$

- (i) By using elementary row operations, reduce its augmented matrix to the following echelon form

$$\left[\begin{array}{ccc|c} 1 & -3 & 2 & 2 \\ 0 & 1 & 4 & * \\ 0 & 0 & * & * \end{array} \right]$$

where '*' represents an expression in β .

[3 marks]

- (ii) Find the values of β for which the system has unique solution and infinitely many solutions. In each case justify your answer with rank test.

[4 marks]

- (b) Suppose that the augmented matrix for a linear system (with variables x, y, z, w) has been row-reduced to the given REF below. Solve the system and express your answer in parametric form.

$$\left[\begin{array}{cccc|c} 1 & 0 & 8 & -5 & 6 \\ 0 & 1 & 4 & -9 & 3 \\ 0 & 0 & 1 & 1 & 2 \end{array} \right]$$

[5 marks]

(c) Given that $A = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 2 & 0 \\ 4 & 0 & 4 \end{bmatrix}$.

(i) Find A^{-1} using elementary row operations.

[5 marks]

(ii) Use A^{-1} to find the solution vector X to the equation $AX = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$.

[2 marks]

(iii) Without computation, state the solution vector Y to the equation $AY = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$. Justify your answer.

[3 marks]

(d) Find the rank of the following matrix by using row operations:

$$\begin{bmatrix} 1 & -1 & 3 \\ 2 & 0 & 4 \\ -1 & -3 & 1 \end{bmatrix}$$

[3 marks]

Question 2

(a) Compute $\det[M]$ if

$$\det \begin{bmatrix} a & b & c \\ p & q & r \\ x & y & z \end{bmatrix} = 5$$

and

$$M = \begin{bmatrix} a + 2x & b + 2y & c + 2z \\ 3x + 4p & 3y + 4q & 3z + 4r \\ -2p & -2q & -2r \end{bmatrix}$$

[5 marks]

- (b) Use Cramer's rule to solve for x in the following system.

$$2x - y + 2z = 3$$

$$x + 2y - 4z = 1$$

$$3x - 4y + z = 2$$

Do NOT solve for y and z .

[4 marks]

(c) Let $A = \begin{bmatrix} 1 & -1 & 8 \\ 0 & 0 & 6 \\ 0 & -1 & 5 \end{bmatrix}$.

- (i) Find the eigenvalues of A .

[4 marks]

- (ii) Find an eigenvector corresponding to the smallest eigenvalue.

[6 marks]

- (d) Consider the following system of linear equations :

$$5x - y + 2z = 3$$

$$x + 4y - z = 2$$

$$2x + y - 5z = 1$$

- (i) Set up Gauss-Seidel scheme for the system.

[3 marks]

- (ii) Compute **two (2)** iterations, starting with the initial guess

$$x^{(0)} = 1, \quad y^{(0)} = 1, \quad z^{(0)} = 1.$$

Keep 4 decimal places in all calculations.

[3 marks]

Question 3

- (a) Given that $\phi = 2x^2y - 3y^2z$.

- (i) Find the directional derivative of ϕ at $Q(1, 2, -1)$ in the direction of the vector

$$\mathbf{A} = 2\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}.$$

[7 marks]

- (ii) In what direction from Q is the directional derivative maximum? Find the magnitude of this maximum value.

[2 marks]

(b) Given that

$$\mathbf{A} = 3xyz^2\mathbf{i} + 2xy^3\mathbf{j} - x^2yz\mathbf{k}$$

$$\phi = 3x^2 - yz.$$

Find the following at the point $(1, -1, 1)$:

(i) $\nabla \cdot \mathbf{A}$

[3 marks]

(ii) $\mathbf{A} \cdot \nabla \phi$

[3 marks]

(c) Evaluate $I = \int_A^B (x - y^2) dx + (y + x^2) dy$ along

(i) a straight line from $A(0, 1)$ to $B(1, 2)$,

[4 marks]

(ii) the parabola $x = t, y = t^2 + 1, t \in [0, 1]$.

[4 marks]

(iii) Is I independent of path? Justify your answer.

[2 marks]

Question 4

(a) Consider the vector field $\mathbf{F} = 2xy\mathbf{i} + (x^2 - 3y^2)\mathbf{j} + 2z\mathbf{k}$.

(i) Show that \mathbf{F} is conservative.

[2 marks]

(ii) Find the scalar field ϕ such that $\mathbf{F} = \nabla \phi$.

[4 marks]

(iii) Evaluate the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$ where C is a simple path joining the point $P(0, 0, 0)$ to the point $Q(1, 1, 1)$.

[2 marks]

(b) Consider the line integral

$$\oint_C (x + y)dx + x^2 dy$$

where C is the triangular path OPQ defined by $O(0, 0)$, $P(1, 0)$ and $Q(0, 1)$ in the counter-clockwise direction. Evaluate the integral

(i) by direct method,

[5 marks]

(ii) by Green's theorem.

[5 marks]

- (c) Use Stokes' theorem to evaluate $\iint_S \nabla \times \mathbf{F} \cdot \mathbf{n} \, dS$ for the function

$$\mathbf{F} = -y\mathbf{i} + x\mathbf{j} + z\mathbf{k}$$

where S is the hemisphere $S : x^2 + y^2 + z^2 = 4, z \geq 0$ and \mathbf{n} is the outward pointing unit normal vector to the surface S .

[7 marks]

Question 5

- (a) A double integral is given by

$$\int_0^1 \int_{x^2}^{2-x} g(x, y) \, dy \, dx.$$

- (i) Sketch and label the region of integration. [2 marks]

- (ii) Hence, rewrite the limits of the integral if the order of integration is reversed. [2 marks]

- (b) Use the Divergence theorem to find $\iint_S \mathbf{F} \cdot d\mathbf{S}$ where

$$\mathbf{F} = x^2\mathbf{i} + 3y^2\mathbf{j} + z^3\mathbf{k}$$

and S is the surface of the cube enclosed by the planes

$$x = \pm 1, y = \pm 1, \text{ and } z = \pm 1. \quad [10 \text{ marks}]$$

- (c) Determine whether each of the following function is either even, odd, or neither.

- (i) $\sin(2x) \cos(x)$ [1 mark]

- (ii) $x^3 \sin(x)$ [1 mark]

- (iii) $x^2 e^{2x}$ [1 mark]

- (d) The periodic function $g(x)$ of period π is defined over the interval $0 < x < \pi$ by

$$g(x) = x, \quad 0 < x \leq \pi.$$

- (i) Sketch the graph of $g(x)$ from -2π to 2π . [2 marks]

- (ii) Show that the Fourier series of $g(x)$ is as follows:

$$g(x) = \frac{\pi}{2} - 2 \left[\frac{1}{2} \sin 2x + \frac{1}{4} \sin 4x + \frac{1}{6} \sin 6x + \dots \right] \quad [6 \text{ marks}]$$

End of Paper

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