

FINAL
Examination Paper
(COVER PAGE)

Session : August 2016

Programme : Diploma In Electrical And Electronic Engineering (DEEI)

Course : EEE2104: Electromagnetic Field Theory

Date of Examination : 9 December 2016 (Friday)

Time : 11:00am – 1:00pm

Duration : 2 Hours Reading Time : Nil

Special Instructions :

This paper consists of SIX (6) questions. Answer any FOUR (4) questions in the answer booklet provided. All questions carry equal marks.

IMPORTANT NOTE : THIS PAPER SHOULD NOT BE TAKEN OUT OF THE EXAMINATION HALL

Materials Permitted : Non Programmable Scientific Calculator

Materials Provided : Appendix

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Moderator : Dr. Ooi Beng Lee

This paper consists of 7 printed pages, including the cover page.

INTI INTERNATIONAL COLLEGE

DIPLOMA IN ELECTRICAL AND ELECTRONIC ENGINEERING (DEEI)
 EEE2104: ELECTROMAGNETIC FIELD THEORY
 FINAL EXAMINATIONS: AUGUST 2016 SESSION

Instructions: This paper consists of **SIX (6)** questions. Answer any **FOUR (4)** questions in the answer booklet provided. All questions carry equal marks. The marks allocated to each sub-question are shown in the brackets at the right-hand margin.

Question 1

- a. Four point charges, each $20\mu\text{C}$, are on the x and y axes at $\pm 4\text{m}$. Find the force on a $100\mu\text{C}$ point charge at $(0, 0, 3)\text{m}$. (7 marks)
- b. A thin disc of radius 5m carries a uniform surface charge density $0.2\mu\text{C}/\text{m}^2$ is placed on the xy -plane with axis same as the z -axis as shown in Figure Q1(a) below. Find the electric field at $(0, 0, 5)$ due to the disc. (9 marks)

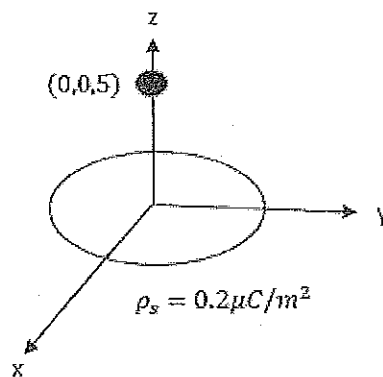


Figure Q1(a)

- c. Determine \bar{D} and \bar{E} at $(2, 0, 2)\text{m}$ due to three standard charge distributions as follows:
- A uniform sheet at $x = 0$ with $\rho_s = \frac{1}{3\pi}\text{nC}/\text{m}^2$
 - A uniform sheet at $x = 4$ with $\rho_s = -\frac{1}{3\pi}\text{nC}/\text{m}^2$
 - A uniform line at $x = 6, y = 0$ with $\rho_l = -2\text{nC}/\text{m}$
- (9 marks)

Question 2

- a. Find the work done in moving a point charge of $5\mu\text{C}$ from the origin to $(2, \pi/4, \pi/2)$ in spherical coordinates, in the field

$$\vec{E} = 5e^{-r/4}\vec{a}_r + \frac{10}{r \sin \theta} \vec{a}_\phi$$

using the path shown in Figure Q2(a) below.

(9 marks)

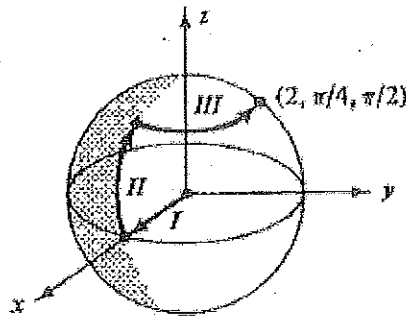


Figure Q2(a)

- b. 40nC of charge is uniformly distributed around a circular ring of radius 2m . Find the potential at a point on the axis 5m from the plane of the ring. Compare the result where all the charge is at the origin in the form of a point charge. (9 marks)

- c. Region 1, defined by $x < 0$, is a free space while region 2, $x > 0$, is a dielectric material for which $\epsilon_{r2} = 2.4$. From Figure Q2(c), given that

$$\vec{D}_1 = 3\vec{a}_x - 4\vec{a}_y + 6\vec{a}_z \text{ C/m}^2$$

find \vec{E}_2 and the angles θ_1 and θ_2 .

(7 marks)

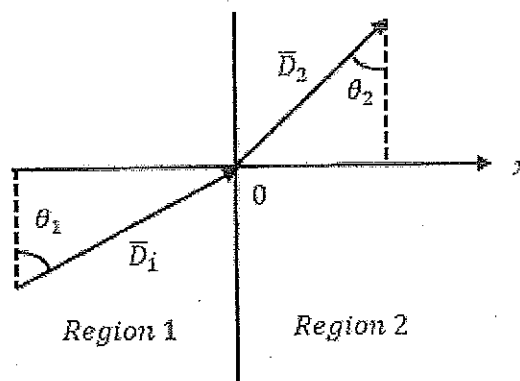


Figure Q2(c)

Question 3

- a. i. Consider length L of two coaxial conductors with inner radius a and outer radius b filled with two different dielectric with relative permittivity ϵ_{r1} and ϵ_{r2} as shown in Figure Q3(a). Prove that the total capacitance is equal to:

(10 marks)

$$C = \frac{\pi \epsilon_0 L}{\ln \frac{b}{a}} (\epsilon_{r1} + \epsilon_{r2})$$

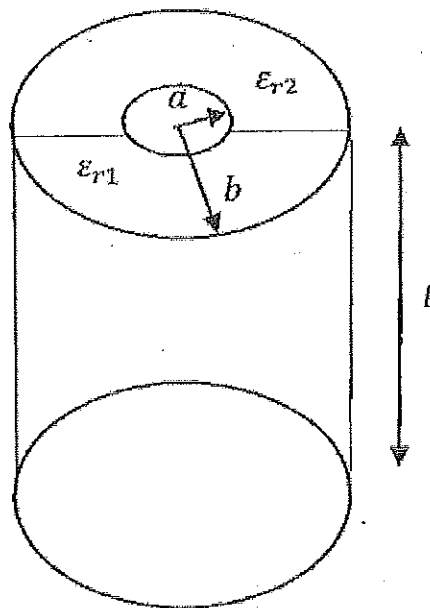


Figure Q3(a)

- ii. Determine the capacitance of 10m length if $a = 1\text{mm}$, $b = 3\text{mm}$, $\epsilon_{r1} = 2.5$, $\epsilon_{r2} = 3.5$. (3 marks)
- b. A free space parallel plate capacitor is charged by momentary connection to a voltage source V , which is then removed. Explain and justify how below parameters change as the plates are moved apart to a separation distance 2 times the original distance.
- The total charge, Q (2 marks)
 - The capacitance, C (2 marks)
 - The electric field intensity, E (2 marks)
 - The electric flux density, D (2 marks)
 - The electric potential, V (2 marks)
 - The energy stored, W (2 marks)

Question 4

- a. Find \vec{H} (in Cartesian Coordinate) at $(0,0,5)$ due to a current line from $x = 0$ to $x = 2$ as shown in Figure Q4(a). (8 marks)

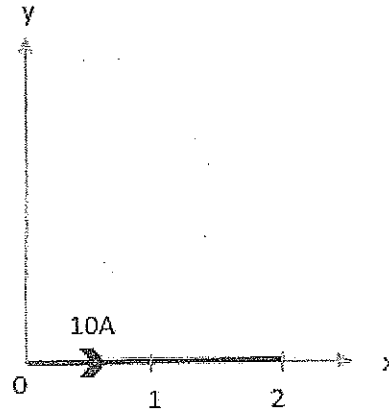


Figure Q4(a)

- b. State Ampere's Law. (2 marks)

- c. An infinitely long solid conductor of radius a is placed along the z -axis. If the conductor carries current I in the $+z$ direction,

- i. show that within the conductor:

$$\vec{H} = \frac{I\rho}{2\pi a^2} \vec{a}_\phi$$

(6 marks)

- ii. find the corresponding current density. (4 marks)

- d. Given that

$$\vec{B} = 2.50 \left(\sin \frac{\pi x}{2} \right) e^{-2y} \vec{a}_z$$

Find the total magnetic flux crossing the surface at $z = 0, y \geq 0, 0 \leq x \leq 2$. (5 marks)

Question 5

- a. A rectangular loop carrying current I_2 is placed parallel to an infinitely long filamentary wire carrying current I_1 as shown in Figure Q5(a) below. Show that the force experienced by the loop is given by

(15 marks)

$$\bar{F} = -\frac{\mu_0 I_1 I_2 b}{2\pi} \left[\frac{1}{\rho_0} - \frac{1}{\rho_0 + a} \right] \bar{a}_\rho$$

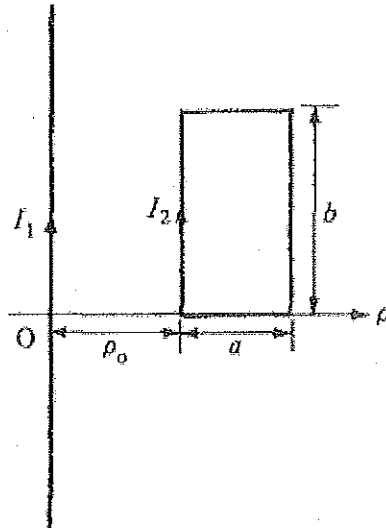
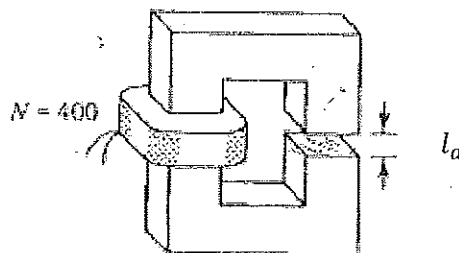


Figure Q5(a)

- b. Define magnetomotive force. (2 marks)
- c. The magnetic circuit shown in Figure Q5(b) is cast iron with a mean length $l_c = 0.44\text{m}$ and square cross section $0.02 \times 0.02\text{m}^2$. The air gap length $l_a = 2\text{mm}$ and the coil contains 400 turns. Find the current required to establish an air gap flux of 0.141mWb . Given that cast iron $\mu = 50\mu_0$ and air $\mu = \mu_0$.

(8 marks)



Question 6

- a. An ideal solenoid has N turns, cross sectional area S and length l . Prove that the self-inductance of the solenoid is equal to (6 marks)

$$L = \frac{\mu N^2 S}{l}$$

- b. In free space,

$$\bar{D} = D_m \sin(\omega t + \beta z) \bar{a}_x$$

Using Maxwell's equations, show that (7 marks)

$$\bar{B} = -\frac{D_m \beta}{\omega \epsilon_0} \sin(\omega t + \beta z) \bar{a}_y$$

- c. A conducting circular loop of radius 20cm lies in the $z = 0$ plane in a magnetic field $\bar{B} = 10 \cos 377t \bar{a}_z \text{ mWb/m}^2$. Calculate the induced voltage in the loop.

(6 marks)

- d. A parallel plate capacitor with plate area of 5cm^2 and plate separation of 3mm has a voltage $50 \sin 10^3 t \text{ V}$ applied to its plate. Calculate the displacement current assuming $\epsilon = 2\epsilon_0$. (6 marks)

~ The End ~
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MATHEMATICAL FORMULAS AND PHYSICAL CONSTANT

Appendix A: Physical Constant

Permittivity of free space, $\epsilon_0 = 8.854 \times 10^{-12} \text{ F/m} \cong \frac{10^{-9}}{36\pi} \text{ F/m}$

Permeability of free space, $\mu_0 = 12.6 \times 10^{-7} \text{ H/m} \cong 4\pi \times 10^{-7} \text{ H/m}$

Appendix B: Trigonometry Identities

Sum or difference of two angles:

$$\sin(a \pm b) = \sin a \cos b \pm \cos a \sin b$$

$$\cos(a \pm b) = \cos a \cos b \mp \sin a \sin b$$

$$\tan(a \pm b) = \frac{\tan a \pm \tan b}{1 \mp \tan a \tan b}$$

Double angle formulas:

$$\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

$$\cos 2\theta = 2 \cos^2 \theta - 1$$

$$\cos 2\theta = 1 - 2 \sin^2 \theta$$

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta$$

Pythagorean Identities:

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\tan^2 \theta + 1 = \sec^2 \theta$$

$$\cot^2 \theta + 1 = \csc^2 \theta$$

Half angle formulas:

$$\sin^2 \theta = \frac{1}{2}(1 - \cos 2\theta)$$

$$\cos^2 \theta = \frac{1}{2}(1 + \cos 2\theta)$$

$$\sin \frac{\theta}{2} = \pm \sqrt{\frac{1 - \cos \theta}{2}}$$

$$\cos \frac{\theta}{2} = \pm \sqrt{\frac{1 + \cos \theta}{2}}$$

$$\tan \frac{\theta}{2} = \pm \sqrt{\frac{1 - \cos \theta}{1 + \cos \theta}} = \frac{\sin \theta}{1 + \cos \theta} = \frac{1 - \cos \theta}{\sin \theta}$$

Appendix C: Differential Length, Area and Volume

Cartesian System :

Differential displacement, $d\bar{l} = dx\bar{a}_x + dy\bar{a}_y + dz\bar{a}_z$

$$d\bar{s}_x = dydz\bar{a}_x$$

Differential normal area, $d\bar{s}_y = dx dz\bar{a}_y$

$$d\bar{s}_z = dx dy\bar{a}_z$$

Differential volume, $dv = dx dy dz$

Cylindrical System :

Differential displacement, $d\bar{l} = d\rho\bar{a}_\rho + \rho d\phi\bar{a}_\phi + dz\bar{a}_z$

Differential normal area, $d\bar{S} = \rho d\phi dz \bar{a}_\rho$

$$= d\rho dz \bar{a}_\phi$$

$$= \rho d\phi d\rho \bar{a}_z$$

Differential volume, $dv = \rho d\rho d\phi dz$

Spherical System :

Differential displacement, $d\bar{l} = dr\bar{a}_r + r d\theta\bar{a}_\theta + r \sin\theta d\phi\bar{a}_\phi$

Differential normal area, $d\bar{S} = r^2 \sin\theta d\theta d\phi \bar{a}_r$

$$= r \sin\theta dr d\phi \bar{a}_\theta$$

$$= r dr d\theta \bar{a}_\phi$$

Differential volume, $dv = r^2 \sin\theta dr d\theta d\phi$

Appendix D: Gradient of scalar

$$\text{Cartesian System} : \nabla V = \frac{\partial V}{\partial x} \bar{a}_x + \frac{\partial V}{\partial y} \bar{a}_y + \frac{\partial V}{\partial z} \bar{a}_z$$

$$\text{Cylindrical System} : \nabla V = \frac{\partial V}{\partial \rho} \bar{a}_\rho + \frac{1}{\rho} \frac{\partial V}{\partial \phi} \bar{a}_\phi + \frac{\partial V}{\partial z} \bar{a}_z$$

$$\text{Spherical System} : \nabla V = \frac{\partial V}{\partial r} \bar{a}_r + \frac{1}{r} \frac{\partial V}{\partial \theta} \bar{a}_\theta + \frac{1}{r \sin \theta} \frac{\partial V}{\partial \phi} \bar{a}_\phi$$

Appendix E: Curl of a Vector

$$\text{Cartesian System} : \nabla \times \bar{A} = \begin{vmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

$$\text{Cylindrical System} : \nabla \times \bar{A} = \frac{1}{\rho} \begin{vmatrix} \bar{a}_\rho & \rho \bar{a}_\phi & \bar{a}_z \\ \frac{\partial}{\partial \rho} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ A_\rho & \rho A_\phi & A_z \end{vmatrix}$$

$$\text{Spherical System} : \nabla \times \bar{A} = \frac{1}{r^2 \sin \theta} \begin{vmatrix} \bar{a}_r & r \bar{a}_\theta & r \sin \theta \bar{a}_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ A_r & r A_\theta & r \sin \theta A_\phi \end{vmatrix}$$

Appendix F: Divergence of a vector

$$\text{Cartesian System} : \nabla \cdot \bar{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\text{Cylindrical System} : \nabla \cdot \bar{A} = \frac{1}{\rho} \frac{\partial}{\partial \rho} (\rho A_\rho) + \frac{1}{\rho} \frac{\partial}{\partial \phi} (A_\phi) + \frac{\partial}{\partial z} (A_z)$$

$$\text{Spherical System} : \nabla \cdot \bar{A} = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 A_r) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (A_\theta \sin \theta) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \phi} (A_\phi)$$

Appendix G: Laplacian of a scalar

Cartesian System:

$$\nabla^2 \bar{A} = \left(\frac{\partial^2 A_x}{\partial x^2} + \frac{\partial^2 A_x}{\partial y^2} + \frac{\partial^2 A_x}{\partial z^2} \right) \bar{a}_x + \left(\frac{\partial^2 A_y}{\partial x^2} + \frac{\partial^2 A_y}{\partial y^2} + \frac{\partial^2 A_y}{\partial z^2} \right) \bar{a}_y + \left(\frac{\partial^2 A_z}{\partial x^2} + \frac{\partial^2 A_z}{\partial y^2} + \frac{\partial^2 A_z}{\partial z^2} \right) \bar{a}_z$$

Appendix H: Relationship between Cartesian and Cylindrical System

$$\rho = \sqrt{x^2 + y^2}, \quad \tan \phi = \frac{y}{x}, \quad z = z$$

or

$$x = \rho \cos \phi, \quad y = \rho \sin \phi, \quad z = z$$

$$\begin{aligned} \bar{a}_x &= \cos \phi \bar{a}_\rho - \sin \phi \bar{a}_\phi & \bar{a}_\rho &= \cos \phi \bar{a}_x + \sin \phi \bar{a}_y \\ \bar{a}_y &= \sin \phi \bar{a}_\rho + \cos \phi \bar{a}_\phi & \bar{a}_\phi &= -\sin \phi \bar{a}_x + \cos \phi \bar{a}_y \\ \bar{a}_z &= \bar{a}_z & \bar{a}_z &= \bar{a}_z \end{aligned}$$

Appendix I: Vector relationship between Cartesian and Spherical System

$$r = \sqrt{x^2 + y^2 + z^2}, \quad \tan \theta = \frac{\sqrt{x^2 + y^2}}{z}, \quad \tan \phi = \frac{y}{x}$$

or

$$x = r \sin \theta \cos \phi, \quad y = r \sin \theta \sin \phi, \quad z = r \cos \theta$$

$$\begin{aligned} \bar{a}_x &= \sin \theta \cos \phi \bar{a}_r + \cos \theta \cos \phi \bar{a}_\theta - \sin \phi \bar{a}_\phi & \bar{a}_r &= \sin \theta \cos \phi \bar{a}_x + \sin \theta \sin \phi \bar{a}_y + \cos \theta \bar{a}_z \\ \bar{a}_y &= \sin \theta \sin \phi \bar{a}_r + \cos \theta \sin \phi \bar{a}_\theta + \cos \phi \bar{a}_\phi & \bar{a}_\theta &= \cos \theta \cos \phi \bar{a}_x + \cos \theta \sin \phi \bar{a}_y - \sin \theta \bar{a}_z \\ \bar{a}_z &= \cos \theta \bar{a}_r - \sin \theta \bar{a}_\theta & \bar{a}_\phi &= -\sin \phi \bar{a}_x + \cos \phi \bar{a}_y \end{aligned}$$

Appendix J: Derivatives of Trigonometric Functions

$$\frac{d}{dx} \sin(x) = \cos(x)$$

$$\frac{d}{dx} \cos(x) = -\sin(x)$$

$$\frac{d}{dx} \tan(x) = \sec^2(x)$$

$$\frac{d}{dx} \cot(x) = -\csc^2(x)$$

$$\frac{d}{dx} \sec(x) = \sec(x) \tan(x)$$

$$\frac{d}{dx} \csc(x) = -\csc(x) \cot(x)$$

Appendix K: Table of Integrals

$$\int \frac{1}{(a^2 + u^2)^{3/2}} du = \frac{u}{a^2 \sqrt{a^2 + u^2}} + C$$

$$\int \frac{u}{(a^2 + u^2)^{3/2}} du = -\frac{1}{\sqrt{a^2 + u^2}} + C$$

$$\int \frac{1}{a^2 + u^2} du = \frac{1}{a} \tan^{-1} \frac{u}{a}$$

$$\int \frac{1}{\sqrt{a^2 + u^2}} du = \ln(u + \sqrt{a^2 + u^2}) + C$$

Appendix L: Vector Identities

Gradient

1. $\vec{\nabla}(f + g) = \vec{\nabla}f + \vec{\nabla}g$
2. $\vec{\nabla}(cf) = c\vec{\nabla}f$, for any constant c
3. $\vec{\nabla}(fg) = f\vec{\nabla}g + g\vec{\nabla}f$
4. $\vec{\nabla}(f/g) = (g\vec{\nabla}f - f\vec{\nabla}g)/g^2$ at points \vec{x} where $g(\vec{x}) \neq 0$.
5. $\vec{\nabla}(\vec{F} \cdot \vec{G}) = \vec{F} \times (\vec{\nabla} \times \vec{G}) - (\vec{\nabla} \times \vec{F}) \times \vec{G} + (\vec{G} \cdot \vec{\nabla})\vec{F} + (\vec{F} \cdot \vec{\nabla})\vec{G}$

Divergence

6. $\vec{\nabla} \cdot (\vec{F} + \vec{G}) = \vec{\nabla} \cdot \vec{F} + \vec{\nabla} \cdot \vec{G}$
7. $\vec{\nabla} \cdot (c\vec{F}) = c\vec{\nabla} \cdot \vec{F}$, for any constant c
8. $\vec{\nabla} \cdot (f\vec{F}) = f\vec{\nabla} \cdot \vec{F} + \vec{F} \cdot \vec{\nabla}f$
9. $\vec{\nabla} \cdot (\vec{F} \times \vec{G}) = \vec{G} \cdot (\vec{\nabla} \times \vec{F}) - \vec{F} \cdot (\vec{\nabla} \times \vec{G})$

Curl

10. $\vec{\nabla} \times (\vec{F} + \vec{G}) = \vec{\nabla} \times \vec{F} + \vec{\nabla} \times \vec{G}$
11. $\vec{\nabla} \times (c\vec{F}) = c\vec{\nabla} \times \vec{F}$, for any constant c
12. $\vec{\nabla} \times (f\vec{F}) = f\vec{\nabla} \times \vec{F} + \vec{\nabla}f \times \vec{F}$
13. $\vec{\nabla} \times (\vec{F} \times \vec{G}) = \vec{F}(\vec{\nabla} \cdot \vec{G}) - (\vec{\nabla} \cdot \vec{F})\vec{G} + (\vec{G} \cdot \vec{\nabla})\vec{F} - (\vec{F} \cdot \vec{\nabla})\vec{G}$

Laplacian

14. $\vec{\nabla}^2(f + g) = \vec{\nabla}^2f + \vec{\nabla}^2g$
15. $\vec{\nabla}^2(cf) = c\vec{\nabla}^2f$, for any constant c
16. $\vec{\nabla}^2(fg) = f\vec{\nabla}^2g + 2\vec{\nabla}f \cdot \vec{\nabla}g + g\vec{\nabla}^2f$

Degree Two

17. $\vec{\nabla} \cdot (\vec{\nabla} \times \vec{F}) = 0$
18. $\vec{\nabla} \times (\vec{\nabla}f) = 0$
19. $\vec{\nabla} \cdot (\vec{\nabla}f \times \vec{\nabla}g) = 0$
20. $\vec{\nabla} \cdot (f\vec{\nabla}g - g\vec{\nabla}f) = f\vec{\nabla}^2g - g\vec{\nabla}^2f$
21. $\vec{\nabla} \times (\vec{\nabla} \times \vec{F}) = \vec{\nabla}(\vec{\nabla} \cdot \vec{F}) - \vec{\nabla}^2\vec{F}$

