



FINAL
Examination Paper

(COVER PAGE)

Session : August 2015

Programme : Diploma in Electrical and Electronic Engineering (DEEI)

Course : MAT 1123: Engineering Mathematics 3

Date of Examination : 7th December 2015 (Monday)

Time : 2:00pm – 4:00pm Reading Time : Nil

Duration : 2 Hours

Special Instructions :

This paper consists of FIVE (5) questions. Answer any FOUR (4) questions in the answer booklet provided. All questions carry equal marks.

IMPORTANT NOTE : THIS PAPER SHOULD NOT BE TAKEN OUT OF THE EXAMINATION HALL

Materials permitted :

Non-programmable calculator

Materials provided :

Formula Booklet 1

Examiner(s) : Mr. Chan Ah Wah

Moderator : Dr. Ch'ng Pei Eng

This paper consists of 6 printed pages, including the cover page.

INTI INTERNATIONAL COLLEGE PENANG
 DIPLOMA IN ELECTRICAL AND ELECTRONIC ENGINEERING (DEEI)
 MAT1123 ENGINEERING MATHEMATICS 3
 FINAL EXAMINATION : AUGUST 2015 SESSION

Instructions

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Question 1

- (a) Consider the following system of linear equations where β is a real constant.

$$\begin{aligned} x - 3y + 2z &= 2 \\ -2x + 7y &= \beta^2 \\ 3x - 10y + \beta z &= -2 \end{aligned}$$

- (i) By using elementary row operations, reduce the augmented matrix to the following echelon form.

$$\left[\begin{array}{ccc|c} 1 & -3 & 2 & 2 \\ 0 & 1 & 4 & \beta^2 + 4 \\ 0 & 0 & \beta - 2 & \beta^2 - 4 \end{array} \right]$$

[3 marks]

- (ii) Find the value of β for which the system has infinitely many solutions. Solve the system for this value of β , and express the solution in vector form .

[7 marks]

- (iii) Find the values of β for which the system has a unique solution. Justify your answer using rank test .

[2 marks]

- (iv) Solve the system for $\beta = 3$.

[5 marks]

(b) Given that $\mathbf{B} = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 2 & 0 \\ 4 & 0 & 4 \end{bmatrix}$.

- (i) Find \mathbf{B}^{-1} using elementary row operations.

[5 marks]

(ii) Use \mathbf{B}^{-1} to find the solution vector \mathbf{X} to the equation $\mathbf{BX} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$.

[3 marks]

Question 2

(a) Compute $\det[\mathbf{M}]$ if

$$\det \begin{bmatrix} a & b & c \\ p & q & r \\ x & y & z \end{bmatrix} = 5$$

and

$$\mathbf{M} = \begin{bmatrix} a+2x & b+2y & c+2z \\ 3x+4p & 3y+4q & 3z+4r \\ -2p & -2q & -2r \end{bmatrix}.$$

[5 marks]

(b) Use Cramer's-rule to solve for x in the following system .

$$2x - y + 2z = 3$$

$$x + 2y - 4z = 1$$

$$3x - 4y + z = 2$$

Do NOT solve for y and z .

[4 marks]

(c) Let $\mathbf{A} = \begin{bmatrix} 1 & -1 & 8 \\ 0 & 0 & 6 \\ 0 & -1 & 5 \end{bmatrix}$.

(i) Find the eigenvalues of \mathbf{A} .

[4 marks]

(ii) Find an eigenvector corresponding to the smallest eigenvalue .

[6 marks]

(d) Consider the following system of linear equations :

$$2x + y - 5z = 1$$

$$x + 4y - z = 2$$

$$5x - y + 2z = 3$$

- (i) Rearrange the given equations and write the system in the form that is diagonally dominant. Hence, set up Gauss-Seidel scheme for the system .

[3 marks]

- (ii) Compute **two (2)** iterations to solve the system, starting with the initial guess

$$x^{(0)} = 1, \quad y^{(0)} = 1, \quad z^{(0)} = 1.$$

Keep 4 decimal places in all calculations .

[3 marks]

Question 3

- (a) Given that $\phi = 2x^2y - 3y^2z$.

- (i) Find the directional derivative of ϕ at $Q(1, 2, -1)$ in the direction of the vector

$$\mathbf{A} = 2\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}.$$

[7 marks]

- (ii) In what direction from Q is the directional derivative maximum? Find also the magnitude of this maximum .

[2 marks]

- (b) Given that

$$\mathbf{A} = 3xyz^2\mathbf{i} + 2xy^3\mathbf{j} - x^2yz\mathbf{k}$$

$$\phi = 3x^2 - yz.$$

Find the following at the point $(1, -1, 1)$:

- (i) $\nabla \cdot \mathbf{A}$

[2 marks]

- (ii) $\mathbf{A} \cdot \nabla \phi$

[4 marks]

- (c) Evaluate $I = \int_A^B (x - y^2) dx + (y + x^2) dy$ along

- (i) a straight line from $A(0, 1)$ to $B(1, 2)$,

[4 marks]

- (ii) the parabola $x = t, y = t^2 + 1, t \in [0, 1]$.

[4 marks]

- (iii) Is I independent of path? Justify your answer .

[2 marks]

Question 4

(a) Consider the vector field $\mathbf{F} = 2xy \mathbf{i} + (x^2 - 3y^2) \mathbf{j} + 2z \mathbf{k}$.

(i) Show that \mathbf{F} is conservative.

[2 marks]

(ii) Find the scalar field ϕ such that $\mathbf{F} = \nabla\phi$.

[4 marks]

(iii) Evaluate the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$ where C is a simple path joining the point $P(0, 0, 0)$ to the point $Q(1, 1, 1)$.

[2 marks]

(b) Consider the line integral

$$\oint_C (x+y)dx + x^2 dy$$

where C is the triangular path OPQ defined by $O(0, 0)$, $P(1, 0)$ and $Q(0, 1)$ in the counter-clockwise direction. Evaluate the integral

(i) by direct method,

[5 marks]

(ii) by Green's theorem.

[5 marks]

(c) Use Stokes' theorem to evaluate $\iint_S \nabla \times \mathbf{F} \cdot \mathbf{n} dS$ for the function

$$\mathbf{F} = -y \mathbf{i} + x \mathbf{j} + z \mathbf{k}$$

where S is the hemisphere $S : x^2 + y^2 + z^2 = 4, z \geq 0$ and \mathbf{n} is the outward pointing unit normal vector to the surface S .

[7 marks]

Question 5

(a) A double integral is given by

$$\int_0^1 \int_{x^2}^{2-x} g(x, y) dy dx.$$

(i) Sketch and label the region of integration.

[2 marks]

(ii) Hence, rewrite the limits of the integral if the order of integration is reversed.

[2 marks]

(b) Use the Divergence theorem to find $\iint_S \mathbf{F} \cdot d\mathbf{S}$ where

$$\mathbf{F} = x^2 \mathbf{i} + 3y^2 \mathbf{j} + z^3 \mathbf{k}$$

and S is the surface of the cube enclosed by the planes

$$x = \pm 1, y = \pm 1, \text{ and } z = \pm 1.$$

[10 marks]

(c) Determine whether each of the following function is either even, odd, or neither.

(i) $\sin(2x) \cos(x)$

[1 mark]

(ii) $x^3 \sin(x)$

[1 mark]

(d) The periodic function $g(x)$ of period π is defined over the interval $0 < x < \pi$ by

$$g(x) = x, \quad 0 < x \leq \pi.$$

(i) Sketch the graph of $g(x)$ from -2π to 2π .

[3 marks]

(ii) Show that the Fourier series of $g(x)$ is as follows :

$$g(x) = \frac{\pi}{2} - 2 \left[\frac{1}{2} \sin 2x + \frac{1}{4} \sin 4x + \frac{1}{6} \sin 6x + \dots \right]$$

[6 marks]

————— End of Paper —————

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