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FINAL
Examination Paper
(COVER PAGE)

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Session : August 2012

Programme : Diploma in Electrical and Electronic Engineering (DEE)

Course : **MAT 1123: Engineering Mathematics 3**

Date of Examination : 14 December 2012

Time : 11a.m. – 1p.m. Reading Time : Nil

Duration : 2 Hours

Special Instructions :

This paper consists of FIVE (5) questions. Answer any FOUR (4) questions in the answer booklet provided. All questions carry equal marks.

Materials permitted :

Non-programmable calculator

Materials provided :

Formula Booklet 1

Examiner(s) : Chan Ah Wah

Moderator : Ch'ng Pei Cheng

This paper consists of 5 printed pages, including the cover page.

INTI INTERNATIONAL COLLEGE PENANG
 DIPLOMA IN ELECTRICAL AND ELECTRONIC ENGINEERING (DEE)
 MAT1123 ENGINEERING MATHEMATICS 3
 FINAL EXAMINATION : AUGUST 2012 SESSION

Instructions

This paper consists of FIVE (5) questions. Answer any FOUR (4) questions in the answer booklet provided. All questions carry equal marks.

Question 1

- (a) Determine which of the following matrices are in ref, rref or neither.

$$(i) \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \quad (ii) \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (iii) \begin{bmatrix} 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

[3 marks]

- (b) Use Gauss-Jordan method to solve the following system of linear equations.

$$\begin{aligned} x_1 - x_2 + x_4 &= 0 \\ -2x_1 + 3x_2 + x_3 - 5x_4 &= 0 \\ 3x_1 + x_2 + x_3 - 2x_4 &= 0 \end{aligned}$$

Express the solution in vector form.

[7 marks]

- (c) For what values of α and β does the following system has

- (i) unique solution,
 (ii) infinitely many solutions,
 (iii) no solution.

$$\begin{aligned} x - 2y + 3z &= 4 \\ 2x - 3y + \alpha z &= 5 \\ 3x - 4y + 5z &= \beta \end{aligned}$$

Justify your answers using rank test.

[9 marks]

- (d) Determine $\text{adj}(A)$ if matrix A is given by

$$A = \begin{bmatrix} 1 & 0 & -2 \\ -3 & 1 & 4 \\ 2 & -3 & 4 \end{bmatrix}.$$

Show all your workings.

[6 marks]

Question 2

(a) Find matrix M if $\left\{ M^T + 2 \begin{bmatrix} 1 & -2 \\ 5 & 0 \end{bmatrix} \right\}^T = 5 \begin{bmatrix} 3 & -1 \\ 1 & 7 \end{bmatrix}$

[4 marks]

- (b) Find A^{-1} using elementary row operations if matrix A is given by

$$A = \begin{bmatrix} 1 & 3 & -1 \\ -1 & 0 & 0 \\ 0 & 2 & -1 \end{bmatrix}$$

[8 marks]

- (c) Use Gauss-Seidel method to solve the following linear system starting with the initial guess $x^{(0)} = 1$, $y^{(0)} = 1$, $z^{(0)} = 1$. Carry out TWO (2) iterations and keep 4 decimal places in all your calculations.

$$5x - y + 2z = 9$$

$$x + 4y - z = 6$$

$$2x + y - 5z = -15$$

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[6 marks]

- (d) Let matrix $A = \begin{bmatrix} 2 & 6 \\ \alpha & \beta \end{bmatrix}$, where α and β are numbers, and A has eigenvectors

$$x_1 = \begin{bmatrix} 3 \\ 1 \end{bmatrix} \quad \text{and} \quad x_2 = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$$

corresponding to eigenvalues λ_1 and λ_2 respectively.

- (i) Find the eigenvalues λ_1 and λ_2 of A by using the property $Ax = \lambda x$.

[5 marks]

- (ii) Hence, determine the values of α and β .

[2 marks]

Question 3

(a) Evaluate the integral $\int_2^5 \int_1^3 (2x + y) dy dx$

[6 marks]

(b) Given $A = 2i + j - k$ and $B = i - 2j + k$.

(i) Find the angle between A and B .

[3 marks]

(ii) Find the unit vector in the direction of A .

[2 marks]

(iii) Find a unit vector perpendicular to both A and B .

[3 marks]

(c) A vector function is given by $F = (2xy - y^4 + 3) i + (x^2 - 4xy^3) j$.

(i) Show that the function is conservative.

[4 marks]

(ii) Based on the result in part (i) above, find a suitable potential function ϕ such that $F = \nabla\phi$.

[7 marks]

Question 4

(a) The boundary of a closed path C is defined by the triangle OAB with coordinates $O(0, 0)$, $A(1, 0)$ and $B(1, 2)$. Evaluate $\int_C x^2 y dx + x dy$

(i) by direct method,

[7 marks]

(ii) by Green's Theorem.

[6 marks]

(b) Determine using Stokes' theorem the integral

$$\iint_S \nabla \times F \cdot n d\sigma$$

where $F = yi - xj$ and S is the hemisphere given by

$$x^2 + y^2 + z^2 = 9, \quad z \geq 0.$$

Hint : Solve the above problem as a line integral around a closed loop.

[12 marks]

Question 5

- (a) Use Gauss' Divergence theorem to evaluate $\iiint_S \mathbf{F} \cdot \mathbf{n} \, dS$ where $\mathbf{F} = xy\mathbf{i} - xz\mathbf{j} + 3yz\mathbf{k}$, and S is the closed surface of the region bounded by the planes

$$z = 0, z = 4 - y, y = 0, y = 2, x = 0, \text{ and } x = 3.$$

[10 marks]

- (b) Classify the following functions as either even, odd, or neither :

(i) $1 + x^2 + 3x^4$

(ii) $\sinh(x)$

(iii) $x^2 + \sin(x)$

(iv) $\cosh(x)$

[4 marks]

- (c) Find the Fourier series for

$$f(t) = \begin{cases} -\frac{1}{2} & -1 < x \leq 0 \\ \frac{1}{2} & 0 < x < 1 \end{cases}$$

$$f(t) = f(t+2)$$

[11 marks]

—— End of Paper ——

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