INTI International College Penang
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FINAL
Examination Paper

(COVER PAGE)

Session	:	**April 2017**
Programme	:	Diploma In Electrical And Electronic Engineering (DEEI)
Course	:	**MAT1136: Engineering Mathematics 3**
Date of Examination	:	4 August 2017 (Friday)
Time	:	8:00am – 10:00am Reading Time : Nil
Duration	:	2 hours
Special Instructions	:	

This paper consists of **FIVE (5)** questions. Answer any **FOUR (4)** questions in the answer booklet provided. All questions carry equal marks.

Materials permitted :

Non-Programmable Calculator

Materials provided :

Formula Booklet 1

Examiner(s) : **Bark Chee Beng**

Moderator : Dr. Ch'ng Pei Eng

*This paper consists of **6** printed pages, including the cover page.*

INTI INTERNATIONAL COLLEGE PENANG

DIPLOMA IN ELECTRICAL AND ELECTRONIC ENGINEERING (DEEI)
MAT1136 : ENGINEERING MATHEMATICS 3
FINAL EXAMINATION: APRIL 2017 SESSION

Instructions: This paper consists of **FIVE (5)** questions. Answer any **FOUR (4)** questions in the answer booklet provided. All questions carry equal marks.

Question 1

(a) Consider the following system of linear equations where k is a real constant.

$$\begin{aligned} x - 2y + 3z &= 2 \\ 2x - y + 3z &= -2 \\ x + y + (k+1)z &= k - 3 \end{aligned}$$

(i) By using elementary row operations, show that its augmented matrix can be reduced to the following echelon form

$$\left[\begin{array}{ccc|c} 1 & -2 & 3 & 2 \\ 0 & 1 & -1 & -2 \\ 0 & 0 & k+1 & k+1 \end{array}\right]$$

(3 marks)

(ii) Determine the value or values of k for which the system has unique solution. Justify your answer with rank test.

(2 marks)

(iii) Determine the value or values of k for which the system has no solution. Justify your answer.

(2 marks)

(iv) Determine the value or values of k for which the system has many solutions. Justify your answer with rank test. Hence, solve the system and express your answer in parametric form.

(6 marks)

(b) Let $\underline{w} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $[A|b]$ be the augmented matrix of the linear system.

$$\begin{aligned} -x + 3y + z &= 1 \\ 2x + 5y &= 3 \\ 3x + y - 2z &= -2 \end{aligned}$$

(i) Find A^{-1} by means of the formula

$$A^{-1} = \frac{1}{\det(A)} adj(A),$$

where *det(A)* and *adj(A)* denotes the determinant of A and the adjoint matrix of A respectively.

(6 marks)

(ii) Find $\underline{w}$ by mean of A^{-1}.

(3 marks)

(iii) Use Cramer's rule to solve for x. [Note : Do NOT solve for y and z.]

(3 marks)

Question 2

(a) Apply the properties of determinant by row operation to show that

$$\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^2 & y^2 & z^2 \end{vmatrix} = (x-y)(y-z)(z-x).$$

(6 marks)

(b) Given that $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = 15$. Find $\begin{vmatrix} a+c & b+d \\ -2a & -2b \end{vmatrix}$.

(3 marks)

(c) Let $A = \begin{bmatrix} -1 & 2 & 2 \\ 2 & 2 & -1 \\ 2 & -1 & 2 \end{bmatrix}$

(i) Find the eigenvalues of A.

(5 marks)

(ii) Find an eigenvector(s) corresponding to the largest eigenvalue.

(5 marks)

(d) Consider the following system of linear equations :

$$\begin{aligned} 4x - y - z &= 3 \\ -2x + 6y + z &= 9 \\ -x + y + 7z &= -6 \end{aligned}$$

(i) Set up Gauss-Seidel scheme for the system.

(3 marks)

(ii) Using the above scheme, compute two (2) iterations, starting with initial guess

$x^{(0)} = 0,\ y^{(0)} = 0,\ z^{(0)} = 0.$

Keep 3 decimal places in all calculations.

(3 marks)

Question 3

(a) Given that $\phi = \phi(x, y, z) = x^2 y^3 z$

(i) Find the directional derivative of ϕ at Q(1,1,-1) in the direction of the vector $A = -i - 3j + k$.

(6 marks)

(ii) In what direction from Q is the directional derivative maximum? Find the magnitude of this maximum value.

(2 marks)

(b) Given that

$D = xy^2 i + (xy + y^2 z) j + xz^2 k$

$\phi = 2x^2 y - 4y^2 z$

Find the following at the point (1,2,1):

(i) $\nabla \bullet D$

(3 marks)

(ii) $D \bullet \nabla\phi$

(3 marks)

(c) Evaluate $I = \int (x - y + 3)dx - (x + y + 2)dy$

(i) along a straight line from P(0,2) to Q(2,10),

(5 marks)

(ii) along the parabola $x = t, y = 2t^2 + 2, t \in [0,2]$

(5 marks)

Is I independent of path? Justify your answer.

(1 mark)

Question 4

(a) Consider vector field $F = yzi + xzj + xyk$.

(i) Show that F is conservative.

(2 marks)

(ii) Find scalar field ϕ such that $F = \nabla\phi$.

(4 marks)

(iii) Evaluate the line integral $\int_C F \bullet dr$ where C is a simple path joining the point A(0,1,0) to point B(2,1,3).

(2 marks)

(b) Consider the line integral

$$\oint_C (x^2 + y)dx - xy^2 dy$$

where C is the triangular path OAB defined by O(0,0), A(1,1) and B(0,1) in the counter-clockwise direction. Evaluate the integral

(i) by direct method,

(5 marks)

(ii) by Green's theorem.

(5 marks)

(c) Use Stokes' theorem to evaluate $\iint_S \nabla \times F \bullet ndS$ for the function

$$F = xzi + yzj + xyk$$

where S is the part of the sphere $x^2 + y^2 + z^2 = 4$ that lies inside the cylider $x^2 + y^2 = 1$ and above the xy-plane.

(7 marks)

Question 5

(a) A double integral is given by

$$\int_0^8 \int_{\frac{y^2}{32}}^{y^{\frac{1}{3}}} xy\,dx\,dy.$$

(i) Sketch and label the region of integration.

(3 marks)

(ii) Hence, rewrite the limits of integral if the order of integration is reversed.

(1 mark)

(iii) Evaluate the integral using the result obtained from (a)(ii).

(3 marks)

(b) Using the Divergence theorem to find $\iint_S F \bullet dS$ where

$$F = x^3\,\underline{i} + y^3\,\underline{j} + z^3\,\underline{k}$$

and S is the surface enclosed by the cylinder $x^2 + y^2 = 9$ and planes $z=0$ and $z=2$.

(7 marks)

(c) Let $f(x)$ be a function such that

$$f(x) = \begin{cases} 1 - x^2, & -1 < x < 1 \\ 0, & otherwise \end{cases}$$

(i) Sketch a graph of $f(x)$ in the interval $-1 < x < 1$.

(1 mark)

(ii) Show that the Fourier series for $f(x)$ in the interval $-1 < x < 1$ is

$$f(x) = \frac{2}{3} + \frac{4}{\pi^2}\left[\cos \pi x - \frac{1}{4}\cos 2\pi x + \frac{1}{9}\cos 3\pi x - \ldots\right].$$

(10 marks)

-- THE END --