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FINAL
Examination Paper

(COVER PAGE)

Session	:	April 2014
Programme	:	Diploma in Electrical and Electronic Engineering (DEEI)
Course	:	MAT 1123: Engineering Mathematics 3
Date of Examination	:	22 JULY 2014
Time	:	5.00pm – 7.00pm Reading Time : Nil
Duration	:	2 Hours
Special Instructions	:	

This paper consists of **FIVE (5)** questions. Answer any **FOUR (4)** questions in the answer booklet provided. All questions carry equal marks.

Materials permitted :

Non-programmable calculator

Materials provided :

Formula Booklet 1

Examiner(s)	:	**Chan Ah Wah**
Moderator	:	**Dr. Ch'ng Pei Eng**

This paper consists of 6 printed pages, including the cover page.

INTI INTERNATIONAL COLLEGE PENANG

DIPLOMA IN ELECTRICAL AND ELECTRONIC ENGINEERING (DEEI)

MAT1123 ENGINEERING MATHEMATICS 3

FINAL EXAMINATION : APRIL 2014 SESSION

Instructions

This paper consists of **FIVE (5)** questions. Answer any **FOUR (4)** questions in the answer booklet provided. All questions carry equal marks.

Question 1

(a) The augmented matrices of three systems of linear equations are given below. In each case, determine rref $[\mathbf{A}|\mathbf{b}]$ and the respective solution(s), where possible.

(i) $$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 3 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \end{array}\right]$$

[2 marks]

(ii) $$\left[\begin{array}{cccc|c} 1 & 2 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

[2 marks]

(iii) $$\left[\begin{array}{cccc|c} 1 & 2 & 3 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

[4 marks]

(b) By using rank test, find the values of α and β such that the system below has **NO** solutions.

$$x + 2y - 5z = 3$$
$$2x - 4y + 6z = \alpha$$
$$3x + \beta y - 5z = 4$$

[7 marks]

(c) (i) Compute the inverse of the following matrix using elementary row operations:

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 9 \end{bmatrix}$$

[8 marks]

(ii) Hence, use the result in (i) to solve the following system of linear equations:

$$\begin{aligned} x + y + z &= 1 \\ x + 2y + 3z &= 0 \\ x + 4y + 9z &= 1 \end{aligned}$$

[2 marks]

Question 2

(a) Given the matrix

$$\mathbf{A} = \begin{bmatrix} 4 & -6 & 2 \\ 0 & -2 & 2 \\ 0 & -1 & 1 \end{bmatrix}.$$

Find the eigenvalues and eigenvectors of $\mathbf{A}$.

[13 marks]

(b) Solve the following system of linear equations using Gauss-Seidel method. Compute two (2) iterations, starting with the initial guess $x_1^{(0)} = 1,\ x_2^{(0)} = 1, x_3^{(0)} = 1$. Keep 4 decimal places in all calculations .

$$\begin{aligned} 8x_1 - x_2 + 2x_3 &= 21 \\ 2x_1 - 11x_2 - x_3 &= 36 \\ x_1 - x_2 + 9x_3 &= 14 \end{aligned}$$

[7 marks]

(c) Show that

$$\det\begin{bmatrix} 1 & 1 & 1 & 1 \\ \alpha & 1 & 1 & 1 \\ \alpha & \alpha & 1 & 1 \\ \alpha & \alpha & \alpha & 1 \end{bmatrix} = (1-\alpha)^3$$

by first reducing the matrix to an upper triangular form through row operations.

[5 marks]

Question 3

(a) Given the points $P(1,-3,1)$, $Q(2,1,0)$ and $R(1,2,2)$.

(i) Find the vectors PQ and PR.

[3 marks]

(ii) Find a unit vector normal to the plane determined by P, Q and R.

[3 marks]

(iii) Find the area of the triangle PQR.

[2 marks]

(iv) Find the equation of the plane PQR.

[2 marks]

(b) Given that

$$\mathbf{A} = 3xyz^2\mathbf{i} + 2xy^3\mathbf{j} - x^2yz\mathbf{k}$$

$$\phi = 3x^2 - yz.$$

Find the following at the point $(1,-1,\ 1)$:

(i) $\nabla \bullet \mathbf{A}$

[3 marks]

(ii) $\mathbf{A} \bullet \nabla\phi$

[3 marks]

(c) Evaluate $I = \int_A^B (x-y^2)\,dx + (y+x^2)\,dy$ along

(i) a straight line from $A(0,1)$ to $B(1,2)$,

[3 marks]

(ii) the parabola $x = t,\ y = t^2 + 1,\ t \in [0,\ 1]$.

[5 marks]

(iii) Is I independent of path? Justify your answer.

[1 mark]

Question 4

(a) Given that $\mathbf{F} = (x^2 + xy^2)\,\mathbf{i} + (y^2 + x^2y)\,\mathbf{j}$.

(i) Show that $\mathbf{F}$ is a conservative vector field.

[3 marks]

(ii) Find the scalar field ϕ such that $\mathbf{F} = \nabla\phi$.

[7 marks]

(iii) Evaluate the line integral $\int_C \mathbf{F} \bullet d\mathbf{r}$ where C is a simple path joining the point $P(0,0,0)$ to the point $Q(2,2,1)$.

[2 marks]

(b) Use Green's theorem to evaluate

$$\oint_C (3x^2 - 8y^2)\,dx + (4y - 6xy)\,dy$$

where C is the boundary of the region R bounded by $y = \sqrt{x}$ and $y = x^2$ in the counterclockwise direction.

[6 marks]

(c) Use line integration to evaluate

$$\iint_S \nabla \times \mathbf{F} \cdot \mathbf{n}\,dS$$

where $\mathbf{F} = y\,\mathbf{i} - x\,\mathbf{j}$ and S is the hemisphere given by $x^2 + y^2 + z^2 = 1, \;\; z \geq 0$.

[7 marks]

Question 5

(a) Use Gauss' Divergence theorem to find $\iint_S \mathbf{F} \bullet d\mathbf{S}$ where

$$\mathbf{F} = (2x - z)\,\mathbf{i} + x^2y\,\mathbf{j} - xz^2\,\mathbf{k}$$

and S is the surface of the cube completely enclosed by the planes

$$x = 0,\; x = 1,\; y = 0,\; y = 1,\; z = 0 \text{ and } z = 1.$$

[9 marks]

(b) A periodic function is given by

$$f(x) = \begin{cases} -x, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$$

$$f(x) = f(x + 2\pi).$$

(i) Sketch the graph of $f(x)$ for $-3\pi < x < 3\pi$.

[3 marks]

(ii) Derive the first five (5) non-zero terms of the Fourier series for $f(x)$.

[13 marks]

End of Paper

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