INTI INTERNATIONAL UNIVERSITY

FOUNDATION IN SCIENCE (CFSI)
MAT 1211 : MATHEMATICS 2
FINAL EXAMINATION : MAY 2014 SESSION

Instructions : This paper consists of **FIVE (5)** questions. Answer any **FOUR (4)** questions in the answer booklet provided. All questions carry equal marks.

Question 1

(a) **Table A** shows the magnitudes and directions of two vectors **M** and **N**.

Table A

Vector.	Magnitude in pounds (Ib).	Direction in degrees $\left(\vartheta^0\right)$.
M	20 Ib.	$\vartheta_M = 60^0$.
N	30 Ib.	$\vartheta_N = 135^0$.

(i) Find the horizontal and vertical components of the two vectors **M** and **N**.
(4 marks)

(ii) Find the magnitude and direction $\left(0^0 \le \theta \le 360^0\right)$ of the resultant vector.
(2 marks)

(b) Given that the definite integral is $\int_0^2 \sqrt{4+x^2}\, dx$.

By using the iterative method of Simpson's rule for four equally spaced intervals, find an estimate of the definite integral, giving your answer to two decimal places. Show your working clearly.
(7 marks)

(c) Given that the differential equation is $2xy\dfrac{dy}{dx} = x^2 + y^2$.

(i) By using the substitution $y = vx$, where v is a function of x, show that the differential equation can be reduced to the form $x\dfrac{dv}{dx} = \dfrac{1-v^2}{Av}$ where A is an integer.
(4 marks)

(ii) Show that the general solution is $x = B\left(x^2 - y^2\right)$ where B is an arbitrary constant.
(6 marks)

(iii) Given that $y = 2$ when $x = 1$, find the particular solution.
(2 marks)

Question 2

(a) Given that the equation of a curve is $2z^2 = 8 - 2x^2 - 8y^2$.

(i) Express the equation of the curve in the form $\left(\frac{z}{a}\right)^2 + \left(\frac{x}{b}\right)^2 + \left(\frac{y}{c}\right)^2 = 1$ where a, b and c are integers.

(2 marks)

(ii) Hence, identify the name of the curve.

(1 mark)

(b) If $z = \sqrt{x^2 + y^3}$ and x is increasing at rate of $3\frac{units}{\sec}$ and y is increasing at rate of $3\frac{unit}{\sec}$. Find the rate of change of z at the instant when $x = 1$ and $y = 2$.

(6 marks)

(c) Given that the differential equation is $x^2 \frac{dy}{dx} = \tan y$.

(i) Show that the general solution is $\sin y = Ae^{\frac{-1}{x}}$ where A is an arbitrary constant.

(6 marks)

(ii) Show that the particular solution $\sin y = e^{\left(\frac{1}{2} - \frac{1}{x}\right)}$ such that $y = \frac{\pi}{2}$ when $x = 2$.

(2 marks)

(d) The probability that Sarah chosen as a school prefect is $\frac{3}{5}$ and the probability that Aini chosen as a school prefect is $\frac{7}{12}$.

(i) Draw a tree diagram showing all the possible outcomes and their respective probabilities .

(2 marks)

Find the probability that

(ii) neither of them is chosen as a school prefect ,

(2 marks)

(iii) only Sarah is chosen as a school prefect .

(2 marks)

(iv) only one of them is chosen as a school prefect.

(2 marks)

Question 3

(a) **Table B** shows the distribution of marks obtained by 40 students in a test.

Table B

Marks	1 – 10	11 – 20	21 – 30	31 – 40	41 – 50
Number of students (f)	6	9	14	7	4

(i) Copy and complete **Table C**

Table C

Marks	Cumulative Frequency	fm (m: midpoint)	fm^2
0.5 – 10.5			
10.5 – 20.5			
20.5 – 30.5			
30.5 – 40.5			
40.5 – 50.5			
		$\sum fm =$	$\sum fm^2 =$

(3 marks)

Hence, calculate the

(ii) median,

(2 marks)

(iii) mean,

(2 marks)

(iv) variance of the distribution.

(2 marks)

(b) Given that the differential equation is $x\frac{dy}{dx}-2y=\frac{x^2}{x+1}$.

(i) Find the integrating factor of the differential equation.

(4 marks)

(ii) Show that the general solution of the differential equation is $e^{\frac{y}{x^2}}=\frac{Cx}{x+1}$ where C is an arbitrary constant.

(10 marks)

(iii) Show that the particular solution of the differential equation is $\frac{2ex}{x+1}=e^{\frac{y}{x^2}}$ such that $y=1$ when $x=1$.

(2 marks)

Question 4

(a) Given that the system of linear equations is

$$2x+y-5z=-11$$
$$x-y+z=6$$
$$4x+2y-3z=-8.$$

Find the value of y.

(5 marks)

(b) Given that the differential equation is $\frac{d^2y}{dx^2}-3\frac{dy}{dx}+2y=\sin x$.

Find the

(i) complementary function,

(3 marks)

(ii) particular integral,

(9 marks)

(iii) particular solution of the differential equation such that $\frac{dy}{dx}=0$ when $x=0$ and $y=0$.

(8 marks)

Question 5

(a) Given that a complex number is $z = a + bi$, where a and b are real numbers.

(i) If $\frac{a}{2-i} + \frac{b}{2+i} = 2 + 5i$ find the values of a and b.

(7 marks)

If $z_1 = z^2 - 2z$, find

(ii) z_1, giving your answer in rectangular form,

(2 marks)

(iii) the modulus and argument of z_1 (in terms of degree).

(2 marks)

(b) Given that $f(x) = e^x \cos x$.

(i) Evaluate $f(0)$, $f'(0)$, $f''(0)$ and $f'''(0)$.

(10 marks)

(ii) Hence, by using Maclaurin's expansion, show that the first three non-zero terms of $f(x)$ can be expressed as $1 + x - \frac{x^3}{a}$ where a is an integer.

(2 marks)

(iii) By using the result obtained from part (ii), evaluate $\int_0^1 f(x)\,dx$.

(2 marks)

--THE END—