

IINTI INTERNATIONAL UNIVERSITY  
FOUNDATION IN SCIENCE (CFSI)  
MAT 1210: MATHEMATICS 1  
FINAL EXAMINATION: MAY 2014 SESSION

**Instructions:** This paper consists of **FIVE (5)** questions. Answer any **FOUR (4)** questions in the answer booklet provided. All questions carry equal marks.

**Question 1**

(a) Simplify the expression of  $\frac{125^{p+2}}{3^{-3p} \times 15^{3p+2} \times 25}$ . (4 marks)

(b) Solve the following equations for  $x$ .

(i)  $4^{(2x-1)^2} = 2^{x+1}$ . (4 marks)

(ii)  $\sqrt{x-1} = 4$ . (2 marks)

(c) Solve the equations:

(i)  $\log_3 x = 2 - \log_3(x+8)$ . (5 marks)

(ii)  $\log_3 9x = \frac{1}{2} - \log_9 x$ . (5 marks)

(d) The expression  $x^3 + ax^2 + bx - 10$  has a factor  $x - 2$  and leaves a remainder of 5 when divided by  $x + 3$ . Find the values of  $a$  and  $b$ . (5 marks)

**Question 2**

- (a) Given that the function  $f(x) = \frac{h}{x-k}$ ,  $x \neq k$ .
- (i) If  $f(4) = -10$  and  $f(10) = 2$ , find the values of  $h$  and  $k$ . (5 marks)
- (ii) Hence, if  $f(m) = m$ , find the values of  $m$ . (5 marks)
- (b) The function  $f(x) = 3x - a$ , where  $a$  is a constant, is defined for all real numbers  $x$ . In the case where  $a = 1$ ,
- (i) solve the equation  $ff(x) = 5$ . (3 marks)
- (ii) The function  $g(x) = 2x^2 + 15x - 33$  is defined for all real numbers  $x$ . Find the values of  $x$  if  $f(x) = g(x)$ . (3 marks)
- (c) (i) Express  $h(x) = x^2 - 8x$  in the form of  $(x - p)^2 - q$  where  $p$  and  $q$  are constants. (2 marks)
- (ii) Hence, find an expression for  $h^{-1}(x)$ . (2 marks)
- (d) Given the two points are  $A(72, 9)$  and  $B(36, -18)$ . Find
- (i) the gradient of the straight line joining the pair of points, (2 marks)
- (ii) the equation of the straight line passing through the pair of points and gives your answer in the form  $y = mx + c$ . (3 marks)

**Question 3**

(a) Evaluate the indefinite integral of  $\int e^{-x} \cos x \, dx$ .

(5 marks)

(b) Prove the following trigonometric identities:

(i) 
$$\frac{\sin 3x}{\sin x} + \frac{\cos 3x}{\cos x} = 4 \cos 2x.$$

(5 marks)

(ii) 
$$\frac{\tan x + \tan y}{\tan x - \tan y} = \frac{\sin(x + y)}{\sin(x - y)}.$$

(4 marks)

(c) Solve the following equation for values of  $\theta$  between  $0^\circ$  and  $360^\circ$  inclusive.

$$\sec^2 \theta + 2 \tan \theta - 6 = 0.$$

(5 marks)

(d) In studying the interference of light waves, the identity  $\frac{\sin\left(\frac{3}{2}x\right)}{\sin\left(\frac{1}{2}x\right)} \sin x = \sin x + \sin 2x$  is

used. Prove this identity.

(6 marks)

**Question 4**

(a) Differentiate the following functions with respect to  $x$ .

(i)  $y = 2\sin^4 x$  (3 marks)

(ii)  $y = \frac{\cos^3 2x}{2}$  (3 marks)

(iii)  $y = e^{-2x} \cos 2x$  (3 marks)

(iv)  $y = 2x \ln 2x$  (3 marks)

(b) Given the equation of a curve is  $y = x^3 - 2x^2 + x$ . Find

(i)  $\frac{dy}{dx}$ , (1 mark)

(ii)  $\frac{d^2y}{dx^2}$ , (1 mark)

(iii) the coordinates of the turning point. (4 marks)

(iv) the local maximum and local minimum. (2 marks)

(c) The volume,  $V \text{ cm}^3$ , of the liquid in the bucket, when the depth of liquid is  $x \text{ cm}$ , is given by

$$V = 0.01x^3 + 2.2x^2 + 200x$$

Find the depth of liquid when the liquid is poured into a bucket at a rate of  $60 \text{ cm}^3 / \text{sec}$  and the rate of increase for the depth of liquid is  $0.2 \text{ cm} / \text{sec}$ .

(5 marks)

**Question 5**

- (a) Evaluate  $\int_{\frac{\pi}{2}}^{\pi} \cos^2 x \, dx$  (4 marks)
- (b) (i) Find the  $x$ -coordinates of the points of intersection between two curves  $y = x^3$  and  $y = 3x^2$ . (2 marks)
- (ii) Find the area of the region enclosed by the two curves  $y = x^3$  and  $y = 3x^2$ . (4 marks)
- (c) (i) Find the  $x$ -coordinates of the points of intersection between the line  $y = 2$  and the two curves  $y = x^2 + 1$ . (2 marks)
- (ii) Find the volume of the region enclosed by the curve and the line in part (i) which is rotated through  $360^\circ$  or  $2\pi$  about the  $x$ -axis. (5 marks)
- (d) Simplify  $\frac{x^{-\frac{1}{3}} y^{\frac{1}{6}} z^{-2} y^{-\frac{1}{2}}}{(x^2 y^4)^{-\frac{1}{6}}}$  (4 marks)
- (e) Find the values of  $\frac{dy}{dx}$  for the function of  $y = (x - 1)^3 (x + 3)^2$  when  $x = 1$ . (4 marks)

**--THE END--**