INTI INTERNATIONAL UNIVERSITY

FOUNDATION IN SCIENCE (CFSI)
MAT 1211 : MATHEMATICS 2
FINAL EXAMINATION : MAY 2014 SESSION

Instructions : This paper consists of **FIVE (5)** questions. Answer any **FOUR (4)** questions in the answer booklet provided. All questions carry equal marks.

Question 1

(a) Given that the equation of a curve is $36z = 4y^2 - 9x^2$.

(i) Express the equation of the curve in the form $z = \left(\frac{y}{a}\right)^2 - \left(\frac{x}{b}\right)^2$ where a and b are integers.

(2 marks)

(ii) Hence, identify the name of the curve.

(1 mark)

(b) Given that the definite integral is $\int_0^4 \frac{1}{\sqrt{4+x^3}}\,dx$.

By using the iterative method of Trapezium's rule for four equally spaced intervals, find an estimate of the definite integral, giving your answer to two decimal places. Show your working clearly.

(7 marks)

(c) Given that the differential equation is $xy\frac{dy}{dx} = x^2 - y^2$.

(i) By using the substitution $y = vx$, where v is a function of x, show that the differential equation can be reduced to the form $x\frac{dv}{dx} = \frac{1 - Av^2}{v}$ where A is an integer.

(5 marks)

(ii) Show that the general solution is $B = x^2\left(x^2 - 2y^2\right)$ where B is an arbitrary constant.

(8 marks)

(iii) Given that $y = 2$ when $x = 1$, find the particular solution.

(2 marks)

Question 2

(a) **Table A** shows the distribution of marks obtained by 56 students in a test.

Table A

Marks	10 – 19	20 – 29	30 – 39	40 – 49	50 – 59
Number of students (f)	1	9	13	18	15

(i) Copy and complete **Table B**

Table B

Marks	Cumulative frequency	m midpoint	fm (m: midpoint)	fm^2
9.5 – 19.5				
19.5 – 29.5				
29.5 – 39.5				
39.5 – 49.5				
49.5 – 59.5				
			$\sum fm =$	$\sum fm^2 =$

(4 marks)

Hence, calculate the

(ii) mean,

(1 mark)

(iii) variance ,

(2 marks)

(iv) median, ,

(2 marks)

(v) third quartile of the test marks.

(2 marks)

(b) If $z = x^3 + y^3 - 2x^2y$ and x is decreasing at rate of $3\dfrac{units}{\text{sec}}$ and y is increasing at rate of $3\dfrac{unit}{\text{sec}}$. Find the rate of change of z at the instant when $x = 1$ and $y = 2$.

(5 marks)

(c) Given that the differential equation is $x\frac{dy}{dx} = \cos^2 y$.

(i) Show that the general solution is $e^{\tan y} = Ax$ where A is an arbitrary constant.

(5 marks)

(ii) Show that the particular solution is $e^{(\tan y - 1)} = x$ such that $y = \frac{\pi}{4}$ when $x = 1$.

(2 marks)

(iii) Hence, find the value of x when $y = \frac{\pi}{3}$, giving your answer to two decimal places.

(2 marks)

Question 3

(a) A biased die and a biased coin are such that $\Pr(a\ six's) = \frac{2}{3}$ and $\Pr(head) = \frac{1}{3}$ respectively. The coin is spun twice and then followed by the die which is rolled once.

(i) Draw a tree diagram, starting with the two throws of the coins, to show all the possible outcomes of these events.

(3 marks)

Find the probability that

(ii) a six is seen,

(3 marks)

(iii) one of the spun of the coin shows a head and the other does not.

(3 marks)

(b) Given that the differential equation is $x\frac{dy}{dx} - y = \frac{x}{x-1}$.

(i) Show that the integrating factor is $\frac{1}{x}$.

(4 marks)

(ii) Show that the general solution of the differential equation is

$$y = x \ln\left|\frac{A(x-1)}{x}\right|$$

where A is an arbitrary constant.

(9 marks)

(iii) Show that the particular solution of the differential equation is

$$y = x\ln\left|\frac{Be^{0.5}(x-1)}{x}\right|$$

such that $y=1$ when $x=2$ where B is an integer .

(3 marks)

Question 4

(a) Given that $f(x)=e^{3x}$ and $g(x)=\cos x^3$.
By using power series, show that

(i) $f(x)=1+3x+\dfrac{9}{a}x^2+....$ where a is an integer,

(2 marks)

(ii) $g(x)=1-\dfrac{x^6}{2}+\dfrac{x^{12}}{b}+...$ where b is an integer.

(2 marks)

(iii) By using the results obtained in (i) and (ii) , find the expansion of $e^{3x}\cos x^3$ in ascending powers of x up to and including the term in x^2.
.

(4 marks)

(iv) Hence, evaluate $\int_0^1 e^{3x}\cos x^3\,dx$.

(2 marks)

(b) Given that the differential equation is $\dfrac{d^2y}{dx^2}+\dfrac{dy}{dx}-2y=2e^{2x}$.

Find the

(i) complementary function,

(3 marks)

(ii) particular integral,

(5 marks)

(iii) general solution,

(1 mark)

(iv) particular solution of the differential equation such that $\dfrac{dy}{dx}=0$ when $x=0$ and $y=0$,

(6 marks)

Question 5

(a) Given that $z = x^3 y^5 + x\cos y$.

Find

(i) $\dfrac{\partial z}{\partial x}$,

(1 mark)

(ii) $\dfrac{\partial z}{\partial y}$,

(1 mark)

(iii) $\dfrac{\partial^2 z}{\partial y^2}$,

(2 marks)

(iv) $\dfrac{\partial^2 z}{\partial y \partial x}$.

(2 marks)

(b) Given that the system of linear equations is

$$\begin{aligned} 3x - y + 4z &= -15 \\ 2x + 5y &= 29 \\ x - 6y - z &= -24. \end{aligned}$$

By using Cramer's Rule, find the value of z.

(6 marks)

(c) Given that a complex number is $z = a + bi$, where a and b are real numbers.

(i) If $\dfrac{a-i}{3+i} - \dfrac{bi}{3-i} = \dfrac{6(1-i)}{5}$ find the values of a and b.

(9 marks)

(ii) Find the modulus and argument of z_1 (in terms of degree).

(2 marks)

(iii) By using De Moivre's theorem, find the value of z^4, giving your answer in rectangular form.

(2 marks)

--THE END—

(MAT1211/May2014/ William Shak /30.06.2014)