

INTI INTERNATIONAL UNIVERSITY

FOUNDATION IN SCIENCE (CFSI)

MAT1211: MATHEMATICS 2

FINAL EXAMINATION: JANUARY 2014 SESSION

Instructions: This paper consists of **FIVE (5)** questions. Answer any **FOUR (4)** questions in the answer booklet provided. All questions carry equal marks.

Question 1

- (a) Table A shows the magnitudes and directions of the two vectors M and N.

Table A

| Vector. | Magnitude in pounds (lb). | Direction in degrees (g^0). |
|---------|---------------------------|---------------------------------|
| M | 40 lb. | $g_M = 150^0$. |
| N | 50 lb. | $g_N = 240^0$. |

- (i) Find the horizontal and vertical components of the two vectors M and N. (4 marks)
- (ii) Find the magnitude and direction ($0^0 \leq \theta \leq 360^0$) of the resultant vector. (2 marks)
- (b) Given that the definite integral is $\int_2^4 \frac{\ln x}{x} dx$.
- By using the iterative method of Trapezium's rule for four equally spaced intervals, find an estimate of the definite integral, giving your answer to three decimal places. Show your working clearly. (7 marks)
- (c) Given that the differential equation is $(x^2 + xy) \frac{dy}{dx} = xy - y^2$.
- (i) By using the substitution $y = vx$, where v is a function of x , show that the differential equation can be reduced to the form $x \frac{dv}{dx} = \frac{Av^2}{1+v}$ where A is an integer. (4 marks)
- (ii) Show that the general solution is $xy = Be^{\frac{x}{y}}$ where B is an arbitrary constant. (6 marks)
- (iii) Given that $y = \frac{1}{2}$ when $x = 2$, find the particular solution. (2 marks)

Question 2

- (a) Given that the equation of a curve is $64z^2 = 64 - 4x^2 - 16y^2$.
- (i) Express the equation of the curve in the form $\left(\frac{z}{a}\right)^2 + \left(\frac{x}{b}\right)^2 + \left(\frac{y}{c}\right)^2 = 1$ where a , b and c are integers. (2 marks)
- (ii) Hence, identify the name of the curve. (1 mark)
- (b) If $z = \sqrt{x^2 - y^2}$ and x is increasing at rate of $2\frac{\text{units}}{\text{sec}}$ and y is decreasing at rate of $3\frac{\text{unit}}{\text{sec}}$. Find the rate of change of z at the instant when $x=5$ and $y=3$. (6 marks)
- (c) Given that the differential equation is $\frac{dy}{dx} + y \sin x = 0$.
Show that the particular solution of the differential equation is $y = Ae^{(\cos x - 1)}$ such that $y=2$ when $x=0$ where A is an integer. (7 marks)
- (d) A bag contains 15 white and 10 yellow discs. Three discs are chosen at random from the bag without replacement.
- (i) Draw a tree diagram showing all the possible outcomes and their respective probabilities. (3 marks)
- Find the probability that
- (ii) only one yellow disc is obtained, (3 marks)
- (iii) the second disc obtained is a white. (3 marks)

Question 3

- (a) The data below shows the number of books sold by a shop per week for eight weeks.
72 75 76 $(p+3)$ 71 73 74 p
Given that the population mean is 75.
Calculate the value of
- (i) p , (2 marks)

(ii) mode, (1 mark)

(iii) median, (1 mark)

(iv) variance of the data. (2 marks)

(b) Given that the system of linear equations is

$$3x - 4y + 2z = -8$$

$$x + 5y - 3z = -2$$

$$5x + 3y - z = -6$$

By using Cramer's Rule, find the value of x . (6 marks)

(c) Given that the differential equation is $x \frac{dy}{dx} + 2y = \sin x$.

(i) Find the integrating factor of the differential equation. (4 marks)

(ii) Show that the general solution of the differential equation is

$$y = \frac{-\cos x}{x} + \frac{\sin x}{x^2} + \frac{C}{x^2}$$

where C is an arbitrary constant.

(5 marks)

(iii) Find the particular solution of the differential equation such that $y=0$ when $x = \frac{\pi}{2}$.

(2 marks)

(iv) Hence find the value of y when $x = \frac{3\pi}{2}$, giving your answer in terms of π .

(2 marks)

Question 4

(a) Given that a complex number is $z = a + bi$, where a and b are real numbers.

(i) If $\frac{a}{1-i} + \frac{b}{1+i} = 1 + 3i$, find the values of a and b .

(6 marks)

If $z_1 = z^2 - 2z$, find

- (ii) z_1 , giving your answer in rectangular form, (2 marks)
- (iii) the modulus and argument of z_1 (in terms of degree). (2 marks)
- (b) Given that the differential equation is $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} = 8e^{-x}$.
- Find the
- (i) complementary function, (3 marks)
- (ii) particular integral, (5 marks)
- (iii) general solution, (1 mark)
- (iv) particular solution of the differential equation such that $\frac{dy}{dx} = 1$ when $x = 0$ and $y = 0$. (6 marks)

Question 5

- (a) Given that $\frac{dy}{dx} = 2 + x + \sin y$ and $y = 0$ at $x = 0$.
- Use the iterative method of Euler's and with a step length of 0.05, find an estimate of y at $x = 0.2$, giving your answer in 3 decimal places. Show your working clearly. (8 marks)
- (b) Given that $f(x) = \ln \cos x$.
- (i) Evaluate $f(0)$, $f'(0)$, $f''(0)$, $f'''(0)$ and $f^{iv}(0)$. (14 marks)
- (ii) Hence, by using Maclaurin's expansion, show that the first two non-zero terms of $f(x)$ is $\frac{-x^2}{a} - \frac{x^4}{b}$, where a and b are integers. (2 marks)
- (iii) By using the result obtained from part (ii), evaluate $\int_0^1 f(x) dx$. (1 mark)

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