

## INTI INTERNATIONAL UNIVERSITY

## FOUNDATION IN SCIENCE(CFSI)

## MAT 1210 : MATHEMATICS 1

## FINAL EXAMINATION : JANUARY 2014 SESSION

**Instructions:** This paper consists of **FIVE (5)** questions. Answer any **FOUR (4)** questions in the answer booklet provided. All questions carry equal marks.

**Question 1**

(a) The function  $f$  is defined by  $f(x) = ax + b$ ,  $x \in \mathfrak{R}$ , where  $a$  and  $b$  are constants. The function  $g$  is defined by  $g(x) = \frac{1}{x-1}$ ,  $x \neq 1$ ,  $x \in \mathfrak{R}$ .

(i) If  $f(1) = 5$  and  $f(2) = 7$ , find the values of  $a$  and  $b$ . (4 marks)

(ii) Write down and simplify expressions for  $f^{-1}(x)$  and  $gf(x)$ . (2 marks)

(iii) Given that  $f^{-1}(x) = gf(x) - 1$ , find the values of  $x$ . (4 marks)

(b) A spherical balloon is inflated at a rate of  $3\text{cm}^3 \text{sec}^{-1}$ . Find the rate of increase of the radius when the radius is  $2\text{cm}$ , giving your answer in terms of  $\pi$ .

$$\left( \text{Volume} = \frac{4}{3}\pi r^3 \right)$$

(4 marks)

(c) Find the area enclosed by the curve  $y = x - \frac{3}{\sqrt{x}}$  and the  $x$ -axis from  $x = 2$  to  $x = 8$ , giving your answer to two decimal places. (4 marks)

(d) Given that  $y = \frac{2x^2 - 4}{x + 3}$ ,

(i) show that  $\frac{dy}{dx} = \frac{a(x^2 + 6x + 2)}{(x + 3)^2}$  where  $a$  is an integer. (4 marks)

(ii) find the value of  $x$  when  $\frac{dy}{dx} = 1$ , giving your answer to two decimal places. (3 marks)

**Question 2**

(a) Solve the equation  $\frac{9}{25}x = \frac{5}{3}x^{-2}$ . (3 marks)

(b) Solve the equation  $10\sec^2 x - 29\tan x = 0$  for  $0^\circ \leq x \leq 360^\circ$ . (5 marks)

(c) Show that  $\int_{\frac{\pi}{12}}^{\frac{\pi}{4}} \cos^2 x dx = \frac{a\pi + b}{24}$  where  $a$  and  $b$  are integers. (6 marks)

(d) Given that the equation of a curve is  $f(x) = -2x^3 - 3x^2 + 36x + 5$ . Find

(i)  $f'(x)$ , (1 mark)

(ii) the coordinates of the turning points, (4 marks)

(iii)  $f''(x)$ , (1 mark)

(iv) the local minimum and the local maximum points, (2 marks)

(v) the values of  $f(-4)$  and  $f(3)$ , (2 marks)

(vi) the absolute minimum and the absolute maximum values in interval  $[-4, 3]$  of the curve. (1 mark)

**Question 3**

(a) Show that  $\frac{\frac{1}{2}\lg 25 + \lg 3 - 2\lg 15}{\lg 225} = \frac{-1}{2}$ . (3 marks)

(b) Solve the equation  $\sqrt{3x-5} - \sqrt{x+2} = 1$ . (7 marks)

- (c) (i) Evaluate  $\int_1^2 x^3 \ln x dx$ , giving your answer to three decimal places.  
(6 marks)
- (ii) Given that  $y = \ln(x \cos x)$ , show that  $\frac{dy}{dx} = \frac{1}{\pi}$  when  $x = \pi$ .  
(4 marks)
- (iii) By using the substitution  $2x = \sin \theta$ , show that  $\int \frac{1}{\sqrt{1-4x^2}} dx = \frac{1}{2} \sin^{-1}(2x) + c$   
where  $c$  is an arbitrary constant.  
(5 marks)

**Question 4**

- (a) Given that  $\tan \theta = \frac{1}{2}$  and that  $\theta$  is acute, evaluate each of the following.
- (i)  $\tan 2\theta$ .  
(2 marks)
- (ii)  $\operatorname{cosec} 2\theta$ .  
(2 marks)
- (iii)  $\sec 2\theta$ .  
(2 marks)
- (b) Express  $\frac{2-6x+10x^2}{(1-3x)(1+x^2)}$  in partial fractions.  
(8 marks)
- (c) Evaluate the following definite integrals, giving your answers to two decimal places.
- (i)  $\int_0^4 \frac{1}{\sqrt{4-x}} dx$ .  
(3 marks)
- (ii)  $\int_{\frac{1}{2}}^1 \frac{e^{-x^2}}{x^3} dx$ .  
(4 marks)
- (iii)  $\int_{\frac{\pi}{4}}^{\pi} \frac{\sec^2 x + 1}{\tan x + x} dx$ .  
(4 marks)

**Question 5**

(a) Given that the equation of a curve is  $x^2 + 4y^2 = 5xy$ .

(i) Show that  $\frac{dy}{dx} = \frac{ax+by}{8y-5x}$  where  $a$  and  $b$  are integers.

(3 marks)

(ii) Find the points on the curve when  $x=1$ .

(2 marks)

(iii) Hence find the values of  $\frac{dy}{dx}$  when  $x=1$ .

(2 marks)

(b) Given that  $y = \frac{\cos x}{1 + \sin x}$ , show that

(i)  $\frac{dy}{dx} = \frac{-1}{1 + \sin x}$ .

(6 marks)

(ii)  $\frac{dy}{dx} = \frac{-2}{3}$  when  $x = \frac{\pi}{6}$ .

(2 marks)

(c) The line  $y = 9 - 3x$  and the curve  $y = \frac{6}{x}$  meet at two points.

(i) Find the  $x$ -coordinates of the points of intersection between the line and the curve.

(3 marks)

(ii) The region enclosed by the line and the curve between these points is rotated through  $360^\circ$  about the  $x$ -axis. Find the volume generated, giving your answer in terms of  $\pi$ .

(7 marks)

**-- THE END --**

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