

## INTI INTERNATIONAL UNIVERSITY

## FOUNDATION IN SCIENCE(CFSI)

## MAT 1210: MATHEMATICS 1

## FINAL EXAMINATION: JANUARY 2014 SESSION

**Instructions:** This paper consists of **FIVE (5)** questions. Answer any **FOUR (4)** questions in the answer booklet provided. All questions carry equal marks.

**Question 1**

- (a) The radius of a spherical bubble is increasing at the rate of  $0.2 \text{ cm/sec}$ . Find the rates of increase of the volume of the bubble when the radius is  $6 \text{ cm}$ , giving your answer in terms of  $\pi$ . (Volume of bubble =  $\frac{4\pi r^3}{3}$ )

(3 marks)

- (b) Given that  $y = \frac{1 + \cos x}{\sin x}$ . Show that  $\frac{dy}{dx} = \frac{-(1 + \cos x)}{\sin^2 x}$ .

(4 marks)

- (c) Given that the equation of a curve is  $x^2 + 4y^2 + 2xy = 4$ .

- (i) Show that  $\frac{dy}{dx} = \frac{-(x+y)}{x+ky}$  where  $k$  is an integer.

(4 marks)

- (ii) Find the points on the curve when  $x = 0$ .

(2 marks)

- (iii) Hence find the values of  $\frac{dy}{dx}$  when  $x = 0$ .

(2 marks)

- (d) Given that  $f(x) = \sqrt{4 - x^2}$ .

- (i) By using the substitution  $x = 2\sin t$ , show that

$$\int f(x) dx = a \left( \sin^{-1} \left( \frac{x}{2} \right) + \frac{x\sqrt{4-x^2}}{4} \right) + c$$

where  $a$  is an integer and  $c$  is an arbitrary constant.

(8 marks)

- (ii) Hence evaluate  $\int_0^2 f(x) dx$ , giving your answer in terms of  $\pi$ .

(2 marks)

**Question 2**

(a) Given that  $\cos\theta = \frac{3}{5}$  and  $\tan\phi = \frac{-8}{15}$  where  $\theta$  is an acute angle and  $\phi$  is a reflex angle. Without using calculator or table, find the values of

(i)  $\sin(\theta + \phi)$ , (2 marks)

(ii)  $\cos(\theta - \phi)$ , (2 marks)

(iii)  $\tan 2\theta$ . (2 marks)

(b) Express  $\frac{3x^2 + x - 2}{(2x-1)(x-2)^2}$  in partial fractions. (7 marks)

(c) Evaluate the following definite integrals, giving your answers to two decimal places.

(i)  $\int_0^4 \frac{2}{\sqrt{2x+1}} dx$ . (4 marks)

(ii)  $\int_0^{\frac{\pi}{2}} \frac{2\cos x}{1 + \sin x} dx$ . (4 marks)

(iii)  $\int_0^1 \frac{xe^{x^2}}{2} dx$ . (4 marks)

**Question 3**

(a) Use binomial expansion to show that the expression  $\frac{2}{\sqrt{1+2x}}$  can be expressed in the form  $2 - 2x + ax^2 + bx^3 + \dots$  where  $a$  and  $b$  are integers. (4 marks)

(b) Given that the equation of a curve is  $y = (x-1)^2(2x+1)^3$

(i) Show  $\frac{dy}{dx} = 2(ax+b)(x-1)(2x+1)^2$  where  $a$  and  $b$  are integers. (5 marks)

- (ii) Hence find the equation of the tangent to the curve at the point  $(0,1)$ . Given your answer in the form of  $y = mx + c$ .

(2 marks)

- (c) Evaluate the following definite integrals, giving your answers in three decimal places.

(i)  $\int_0^{\frac{\pi}{2}} \sin^2 x \cos^3 x \, dx.$

(8 marks)

(ii)  $\int_0^{\pi} 2x \sin 2x \, dx.$

(6 marks)

#### Question 4

- (a) Solve the equation  $2 \cos e^c x - 3 \cot x - 1 = 0$  for  $0^\circ \leq x \leq 360^\circ$ , giving your answers in terms of degree and to two decimal places.

(6 marks)

- (b) Solve the equation  $\sqrt{1-20x} - 2\sqrt{x+1} = 3.$

(8 marks)

- (c) Given that the equation of a curve is  $f(x) = x^3 + 3x^2 - 9x + 2$ . Find

(i)  $f'(x),$

(1 mark)

- (ii) the coordinates of the turning points,

(5 marks)

(iii)  $f''(x),$

(1 mark)

- (iv) the local minimum and the local maximum points,

(2 marks)

- (v) the absolute minimum and the absolute maximum values in interval  $[-4, 2]$  of the curve.

(2 marks)

**Question 5**

- (a) A quadratic function  $g$  is defined by  $g(x) = ax^2 + bx - 5$ ,  $x \in \mathfrak{R}$  where  $a$  and  $b$  are constants .
- (i) If  $g(-1) = 1$  and  $g(1) = -8$ , find the values of  $a$  and  $b$ .  
(4 marks)
- (ii) Hence find the values of  $x$  when  $g(x) = -5$ .  
(2 marks)
- (b) Solve the equation  $\log_4 x + \log_2 x = 3$ .  
(3 marks)
- (c) Given that the line is  $y = -3x + 9$  and the curve is  $y = \frac{6}{x}$ .
- (i) Find the  $x$ -coordinates of the points of intersection between the line and the curve.  
(3 marks)
- (ii) The region enclosed by the line and the curve between these points is rotated through  $360^\circ$  about the  $x$ -axis. Find the volume generated, giving your answer in terms of  $\pi$ .  
(5 marks)
- (d) A quadratic equation is given as  $x^2 - 10 = 0$ .
- (i) Use Newton-Raphson's method to show that the iterative formula can be simplified to the form  

$$x_{r+1} = \frac{x_r^2 + 10}{2x_r}.$$
(4 marks)
- (ii) Taking  $x_1 = 3.5$  as an initial approximation to the root of the quadratic equation, evaluate  $x_2$ ,  $x_3$  and  $x_4$ , giving your answers to four decimal places.  
(3 marks)
- (iii) From the result obtained in part (ii), determine the approximate root of the quadratic equation to three decimal places.  
(1 mark)

**--THE END--**