FINAL
Alternative Assessment

(COVER PAGE)

Session	:	**January 2021**
Programme	:	Foundation in Science (CFSI)
Course	:	**MAT1211: Mathematics 2**
Date of Examination	:	9 March 2021 (Tuesday)
Time	:	9:00am – 11:30am Reading Time : Nil
Duration	:	2 hours + 30 minutes (uploading time)
Special Instructions	:	

This paper consists of **FOUR (4)** questions. Answer **ALL** questions in the foolscap paper.
All questions carry equal marks.

Materials permitted :

Non-Programmable Calculator

Materials provided :

Formula Booklet 1

Examiner(s)	:	**Mr. Teo Chun Yew**
Chief Moderator	:	Ms. Nurul Asyima Zulkeflee

This paper consists of 5 printed pages, including the cover page.

FOUNDATION IN SCIENCE (CFSI)
MAT 1211: MATHEMATICS 2
FINAL ALTERNATIVE ASSESSMENT: JANUARY 2021 SESSION

Instructions: This paper consists of **FOUR (4)** questions. Answer **ALL** questions in the foolscap paper. All questions carry equal marks.

Question 1

(a) The complex numbers z and w satisfy the equations

$$z + (1+i)w = i \text{ and } (1-i)z + iw = 1.$$

Solve the equations for z and w, giving your answers in the form $x + iy$, where x and y are real.

(6 marks)

(b) Given a complex number $v = 1 - \sqrt{3}i$. Use De Moivre Theorem to find the modulus and argument of $\sqrt{v}$. Hence express $\sqrt{v}$ in the exponential form.

(6 marks)

(c) It is given that $\mathbf{A} = \begin{bmatrix} 1 & 4 \\ 3 & 1 \end{bmatrix}$ and $\mathbf{I}$ is 2×2 identity matrix.

(i) Show that $(\mathbf{A} - \mathbf{I})^2 = k\,\mathbf{I}$, for some integer k.

(2 marks)

(ii) Given further that $\mathbf{B} = \begin{bmatrix} 1 & 3 \\ p & 1 \end{bmatrix}$, find the integer p such that $(\mathbf{A} - \mathbf{B})^2 = (\mathbf{A} - \mathbf{I})^2$.

(3 marks)

(d) A company sold 2 cases of product x, 4 cases of product y, and 6 cases of product z for RM62. One case of product x, 3 cases of product y, and 3 cases of product z for RM36. Two cases of product x, 3 cases of product y, and 2 cases of product z for RM39. Use Gaussian elimination method to determine the cost of a case of product x, y, and z respectively.

(8 marks)

Question 2

(a) Given a differential equation

$$\tan x \frac{dy}{dx} = \sqrt{y} + 0.4x^2, \; y(2) = 4.$$

Use Euler method formula to solve the value of y for $x = 2(0.1)2.5$ correct to 2 decimal places. Give the result of each iteration to 4 decimal places.

(6 marks)

(b) Given that $f(x) = \ln(1 + \cos x)$.

(i) Show that $f''(x) = -\frac{1}{1+\cos x}$. (3 marks)

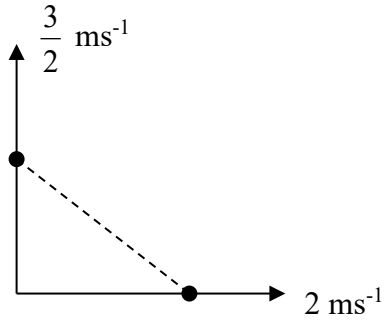
(ii) Hence, find the first two non-zero terms of the Maclaurin series for $f(x)$.

(3 marks)

(c) Given that $z = y^2 e^{xy} - e^y$, find $\frac{\partial z}{\partial y}$, $\frac{\partial^2 z}{\partial y^2}$ and $\frac{\partial^2 z}{\partial x \partial y}$. Give your answer in simplify form.

(6 marks)

(d)



Two men start walking from the same point. One travels *y*-direction at 2 ms^{-1} and the other travels *x*-direction at $\frac{3}{2}$ ms^{-1}. Let *z* be the distance between the two men. Find the rate of change of *z* four seconds later.

(7 marks)

Question 3

(a) The variables x and t satisfy the differential equation

$$e^{-3t}\frac{dx}{dt} = \sec^2 2x,$$

for $t \geq 0$. It is given that $x = \pi$ when $t = 0$.

(i) Solve the differential equation and obtain an expression for x in terms of t. (7 marks)

(ii) Find the value of t when $x = 2\pi$. Leave your answer in 3 decimal places. (2 marks)

(b) By using integrating factors, solve the differential equations

$$(x+1)\frac{dy}{dx} - 3y = (x+1)^2;\ y(0) = 0.$$

Express your answer y in terms of x.

(6 marks)

(c) Two vectors, **u** and **v**, are such that

$$\mathbf{u} = \begin{pmatrix} q \\ 2 \\ 6 \end{pmatrix} \text{ and } \mathbf{v} = \begin{pmatrix} 8 \\ q-1 \\ q^2-7 \end{pmatrix},$$

where q is a constant.

(i) Find the values of q for which **u** is perpendicular to **v**. Hence find the unit vectors of **v** for the $q > 0$. (6 marks)

(ii) Find the angle between **u** and **v** when $q = 0$. (4 marks)

Question 4

(a) The following table shows the range of the ages of 126 students taking a course in a local private college:

Age group (years)	Number of students
18 – 24	7
25 – 31	a
32 – 38	21
39 – 45	b
46 – 52	14

Given that $\sum fx = 4900$. Find the value of a and b. Hence, calculate the mode base on the data.

(8 marks)

(b) Everyday, a fisherman has the choice of fishing at sea, or in a river, or a lake. The probability that he fishes at sea, in the river, and at a lake are $\frac{1}{2}$, $\frac{1}{4}$ and $\frac{1}{4}$ respectively. If he goes out to sea, his will be catching some fish on four days out of five days, while his chances at the river and the lake are only two days out of five days, and three days out of five days respectively.

(i) Draw a tree diagram to represent this information. (3 marks)

(ii) Find the probability that the fisherman catches fish on a randomly-selected day.

(2 marks)

(c) Solve the second order differential equation

$$\frac{d^2y}{dx^2} - 4y = 7x + e^{2x}.$$

Find y in terms of x, given that $y = 2$ and $\frac{dy}{dx} = \frac{1}{2}$ when $x = 0$.

(12 marks)

The End