INTI International University & Colleges

FINAL
Alternative Assessment

(COVER PAGE)

Session	:	**August 2020**
Programme	:	Foundation in Science (CFSI)
Course	:	**MAT1211: Mathematics 2**
Date of Examination	:	14 December 2020 (Monday)
Time	:	9:00am – 11:30am Reading Time : Nil
Duration	:	2 hours + 30 minutes (uploading time)
Special Instructions	:	

This paper consists of **FOUR (4)** questions. Answer **ALL** questions.
All questions carry equal marks.

Materials permitted :

Non-Programmable Calculator

Materials provided :

Formula Booklet 1

Examiner(s)	:	**Mr. Teo Chun Yew**
Chief Moderator	:	Mr. Goh Chok Huat

This paper consists of 5 printed pages, including the cover page.

INTI INTERNATIONAL COLLEGE PENANG

FOUNDATION IN SCIENCE (CFSI)
MAT 1211: MATHEMATICS 2
FINAL ALTERNATIVE ASSESSMENT: AUGUST 2020 SESSION

Instructions: This paper consists of **FOUR (4)** questions. Answer **ALL** questions. All questions carry equal marks.

Question 1

(a) The complex numbers v and w satisfy the equations

$$v + iw = 5 \text{ and } (1+2i)v - w = 3i .$$

Solve the equations for v and w, giving your answers in the form $x+iy$, where x and y are real.

(6 marks)

(b) Given the complex number $\sqrt{3}+i$ is denoted by u. Express u in the form $r(\cos\theta + i\sin\theta)$, where $r > 0$ and $-\pi < \theta \le \pi$. Hence or otherwise, find the modulus and argument of u^5.

(6 marks)

(c) The matrix $\mathbf{A}$ is defined by $\begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix}$.

(i) Given that $\mathbf{A}^2 = \begin{bmatrix} m & 2 \\ 3 & 6 \end{bmatrix}$, find the value of m.

(2 marks)

(ii) Show that $\mathbf{A}^3 - 7\mathbf{A} = n\mathbf{I}$, where n is an integer and $\mathbf{I}$ is the 2×2 identity matrix.

(3 marks)

(d) The matrices $\mathbf{P}$ and $\mathbf{Q}$ are defined in terms of constant k by

$$\mathbf{P} = \begin{bmatrix} 3 & 2 & 1 \\ 1 & -1 & k \\ 5 & 3 & 2 \end{bmatrix} \text{ and } \mathbf{Q} = \begin{bmatrix} 5 & 4 & 1 \\ 3 & k & -1 \\ 7 & 3 & 2 \end{bmatrix}.$$

(i) Express $|\mathbf{P}|$ and $|\mathbf{Q}|$ in terms of k. (4 marks)

(ii) Hence, find the possible exact values of k. Given that $|\mathbf{PQ}| = 16$. (4 marks)

Question 2

(a) Use the trapezium rule with four intervals to find an approximation of

$$\int_0^8 \ln(x+2)\ dx,$$

giving your answer correct to 3 decimal places. Hence find an approximation of

$$\int_0^8 3\ln(x^2+4x+4)\ dx.$$

(7 marks)

(b) Given that $f(x)=\dfrac{e^{2x}+e^{-2x}}{2}$.

(i) Show that $f'(x)+2f(x)=2e^{2x}$. (3 marks)

(ii) By differentiating the result in part (i), find the first two non-zero terms of the Maclaurin series for $f(x)$. Give the coefficients in exact form.

(4 marks)

(c) Given that $z=x^2-y^2+xy\ln\left(\dfrac{y}{x}\right)$, show that $x\dfrac{\partial z}{\partial x}+y\dfrac{\partial z}{\partial y}=2z$.

(6 marks)

(d) If a right circular cone grows in height by $\dfrac{dh}{dt}=1$ cm s^{-1} and in radius by $\dfrac{dr}{dt}=2$ cm s^{-1}, starting from zero, how fast is its volume growing at $t=3$ s?

(5 marks)

Question 3

(a) In a certain chemical process a substance is being formed, and t minutes there are x grams of the substance present. In the process the rate of increase x is given as $\frac{dx}{dt} = k(50-x)^2$, where k is a constant. It is know that when $t = 0$, $x = 0$ and $\frac{dx}{dt} = 5$.

(i) Solve the differential equation and show that $x = 50 - \frac{500}{t+10}$.

(6 marks)

(ii) Hence calculate the mass of the substance when $t = 10$, and find the time taken for the mass to increase from 0 to 45 grams.

(2 marks)

(b) By using integrating factors, solve the differential equations

$$\frac{dy}{dx} + \left(\frac{1+x}{x}\right)y = 1;\ y(1) = 3.$$

Express your answer y in terms of x.

(7 marks)

(c) Relative to an origin O, the position vectors of points A and B are given by

$$\overrightarrow{OA} = \begin{pmatrix} 3 \\ -6 \\ p \end{pmatrix} \text{ and } \overrightarrow{OB} = \begin{pmatrix} 2 \\ -6 \\ -7 \end{pmatrix},$$

and angle $AOB = 90^\circ$.

(i) Find the value of p. (2 marks)

The point C is such that $\overrightarrow{OC} = \frac{2}{3}\overrightarrow{OA}$.

(ii) Find the unit vector in the direction of $\overrightarrow{BC}$. (4 marks)

(iii) Find the angle ABC. Leave your answer in 1 decimal place. (4 marks)

Question 4

(a) A survey was made of the journey times of 63 people who cycle to work in a certain town. The results are summarised in the following cumulative frequency table.

Time (min)	$0 < t \le 15$	$15 < t \le 30$	$30 < t \le 45$	$45 < t \le 60$	$60 < t \le 75$
Frequency	8	16	26	9	4

Calculate base on the data, the mean and median.

(5 marks)

(b) A fair eight-sided die has faces marked 1, 2, 3, 4, 5, 6, 7, 8. The score when the die is thrown is the number on the face the die lands on. The die is thrown twice.

- Event *R* is 'one of the scores is exactly 3 greater than the other score'.
- Event *S* is 'the product of the scores is more than 19'.

(i) Find the probability of *R*. (2 marks)

(ii) Find the probability of *S*. (2 marks)

(iii) Determine whether events *R* and *S* are independent. Justify your answer.

(3 marks)

(c) Solve the second order differential equation

$$\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 6y = 6\sin 2t + 30\cos 2t \ .$$

Find y in terms of t, given that $y = 4$ and $\frac{dy}{dt} = -6$ when $t = 0$.

(13 marks)

The End