INTI International College Penang

FINAL
Examination Paper

(COVER PAGE)

Session	:	August 2019
Programme	:	Foundation In Science (CFSI)
Course	:	**MAT1211: Mathematics 2**
Date of Examination	:	7 December 2019 (Saturday)
Time	:	11:00AM – 1:00PM Reading Time : Nil
Duration	:	2 Hours

Special Instructions :

This paper consists of **FIVE (5)** questions. Answer **any FOUR (4)** questions in the answer booklet provided. All questions carry equal marks.

Materials permitted :

Non-Programmable Scientific Calculator

Materials provided :

Formula Booklet 1

Examiner(s)	:	**Mr. Teo Chun Yew**
Moderator	:	Dr. Ch'ng Pei Eng

This paper consists of 5 printed pages, including the cover page

INTI INTERNATIONAL COLLEGE PENANG

FOUNDATION IN SCIENCE (CFSI)
MAT 1211: MATHEMATICS 2
FINAL EXAM: AUGUST 2019 SESSION

Instructions: This paper consists of **FIVE (5)** questions. Answer any **FOUR (4)** questions in the answer booklet provided. All questions carry equal marks.

Question 1

(a) Express each of the following in the form $a+bi$.

(i) $(i^4+i^9+i^{16})(2-i^5+i^{10}+i^{15})$ (2 marks)

(ii) $\dfrac{(1-4i)^2}{2+3i}$ (3 marks)

(b) Given that $z=\dfrac{\sqrt{3}}{4}+\dfrac{i}{4}$.

(i) Find the modulus and argument of z. (2 marks)

(ii) Use De Moivre's theorem to find $(z)^5$, giving your answer in the form of $a+bi$. (3 marks)

(c) Given that

$$\mathbf{A}=\begin{bmatrix}8 & 5\\ 3 & 2\end{bmatrix}, \mathbf{B}=\begin{bmatrix}p & -3\\ -2 & q\end{bmatrix} \text{ and } \mathbf{C}=\begin{bmatrix}16 & 4\\ -7 & -25\end{bmatrix},$$

find value of p and q such that $\mathbf{A}^{-1}\mathbf{B}=\mathbf{C}$. (4 marks)

(d) (i) Given a 3 by 3 matrix $\mathbf{A}=\begin{bmatrix}1 & 1 & -1\\ 4 & -3 & 2\\ 2 & -2 & -3\end{bmatrix}$, find $\mathbf{A}^{-1}$. (8 marks)

(d) (ii) Hence, or otherwise solve the following linear system

$$\begin{aligned} x+\ y-\ z &= -1 \\ 4x-3y+2z &= 16\,. \\ 2x-2y-3z &= \ 5 \end{aligned}$$

(3 marks)

Question 2

(a) Use the Simpson rule with four intervals to estimate the value of

$$\int_2^3 \frac{1}{x\ln x}\,dx,$$

giving your answer correct to 2 decimal places.

(5 marks)

(b) Use Euler method to solve the values of y for $x = 1(0.2)1.6$ if

$$\frac{dy}{dx} = \sqrt{x} - \ln(y+1),\ y(1) = 0\,.$$

Give your answers correct to **four (4)** decimal places.

(5 marks)

(c) By using series expansions from formula booklet, write the first four non-zero terms of $(1-x)e^{x/2}$.

(5 marks)

(d) Derive $f(x) = x^2 + \ln(x+2)$ using Maclaurin series up to and including the term in x^4. Hence approximate the integration of $\int_1^2 x^2 + \ln(x+2)\,dx$, leave your answer in four decimal places.

(10 marks)

Question 3

(a) Given that $z = \dfrac{xy}{x+y}$ find $\dfrac{\partial^2 z}{\partial x^2}$, $\dfrac{\partial^2 z}{\partial y^2}$ and $\dfrac{\partial^2 z}{\partial x \partial y}$. (7 marks)

(b) Given that $f(x,y) = \sqrt{xy} + x^3 - y^3$, approximate the changes of f as (x,y) varies from (3, 3) to (3.1, 2.7).

(8 marks)

(c) Solve the differential equation $2\dfrac{d^2y}{dx^2} - 5\dfrac{dy}{dx} - 3y = 0$ given that $x = 0$ when $y = 3$ and $\dfrac{dy}{dx} = 2$. Hence find the value of y when $x = 2\ln 3$.

(10 marks)

Question 4

(a) A certain chemical substance dissolves in water at a rate given as follows:

$$\frac{dx}{dt} = kx$$

where x is the amount remained at time t. If the chemical substance starts with the amount of 20 grams and left with 8 grams after 2 hours, find the amount left after 6 hours.

(6 marks)

(b) Solve the following differential equations:

(i) $xy\dfrac{dy}{dx} = 2x^2 + 3y^2$, given that $y = 0$ when $x = 1$, (9 marks)

(ii) $\dfrac{d^2y}{dx^2} + 4y = \sin x$, given that $y = 0$ and $\dfrac{dy}{dx} = 2$, when $x = \dfrac{\pi}{2}$. (10 marks)

Question 5

(a) A packet of candies contains 5 red candies and 14 green candies. Two candies are selected at random without replacement.

(i) Draw a tree diagram to show the possible outcomes. Include the probability on each branch.

(2 marks)

(ii) Find the probability that the two candies are of different colours.

(2 marks)

(iii) Find the probability that at least one of the candies is green.

(2 marks)

(b) The life span (in hours) for a certain type of battery is shown in the following table:

Class Limits	Frequency
200 – 299	11
300 – 399	12
400 – 499	16
500 – 599	23
600 – 699	17
700 – 799	11
800 – 899	10

Determine the mean, median and mode of the life span for the 100 batteries.

(9 marks)

(c) The position vectors of points A and B are $\begin{pmatrix} -3 \\ 6 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} -1 \\ 2 \\ 4 \end{pmatrix}$ respectively, relative to an origin O.

(i) Calculate angle AOB. (5 marks)

(ii) The point C is such that $\overrightarrow{AC} = 3\overrightarrow{AB}$. Find the unit vector in the direction of $\overrightarrow{OC}$.

(5 marks)

The End