

**FINAL**  
Alternative Assessment

(COVER PAGE)

Session : August 2021

Programme : Foundation in Science (CFSI)

Course : **MAT1211: Mathematics 2**

Date of Examination : 7 December 2021 (Tuesday)

Time : 9:00am – 11:30am Reading Time : Nil

Duration : 2 hours + 30 minutes (uploading time)

Special Instructions :

This paper consists of **FOUR (4)** questions. Answer **ALL** questions in the foolscap paper.

All questions carry equal marks.

Materials permitted :

Non-Programmable Calculator

Materials provided :

Formula Booklet 1

Examiner(s) : **Mr. Teo Chun Yew**

Chief Moderator : Mr. Wong Caan Ee

*This paper consists of 5 printed pages, including the cover page.*

FOUNDATION IN SCIENCE (CFSI)  
 MAT 1211: MATHEMATICS 2  
 FINAL ALTERNATIVE ASSESSMENT: AUGUST 2021 SESSION

**Instructions:** This paper consists of **FOUR (4)** questions. Answer **ALL** questions in the foolscap paper. All questions carry equal marks.

**Question 1**

(a) Given that  $z = \frac{i(2-i)-1}{1+i}$ .

(i) Express  $z$  in the form  $a + bi$ .

(3 marks)

(ii) Use De Moivre's theorem to find  $z^3$ , giving your answer in the form of  $a + bi$ .

(5 marks)

(b) Solve the equation  $z^2 = -7 + 24i$  for all the complex roots,  $z$ .

(6 marks)

(c) Given that

$$\mathbf{A} = \begin{pmatrix} 2 & p \\ 3 & q \end{pmatrix}, \mathbf{B} = \begin{pmatrix} -2 & 7 \\ 1 & r \end{pmatrix} \text{ and } \mathbf{C} = \begin{pmatrix} 4 & -2 \\ -6 & 2 \end{pmatrix}$$

find value of  $p$ ,  $q$  and  $r$  such that  $|\mathbf{B} + \mathbf{C}| = 25$  and  $\mathbf{A}(\mathbf{B} + \mathbf{C})^{-1} = \frac{1}{25} \begin{pmatrix} 15 & -4 \\ 20 & -7 \end{pmatrix}$ .

(6 marks)

(d) Given that the system of linear equations is

$$3x + 2y - z = 6$$

$$x + 4y + 3z = 5.$$

$$5x - y + z = 4$$

By using Cramer's Rule, find the value of  $x$ .

(5 marks)

**Question 2**

- (a) Use Euler method to solve the values of
- $y$
- for
- $x = 0(0.1)0.4$
- if

$$\frac{dy}{dx} = \tan(x + y), \quad y(0) = 1.$$

Give your answers correct to **four (4)** decimal places.

(6 marks)

- (b) Derive
- $f(x) = x(1 + \ln(x+1))$
- using Maclaurin series up to and including the term in
- $x^4$
- . Hence approximate the integration of
- $\int_0^{0.5} x(1 + \ln(x+1))dx$
- , leave your answer in four decimal places.

(9 marks)

- (c) Given that
- $z = xy + \ln(x + y)$
- show that
- $x \frac{\partial^2 z}{\partial x^2} + y \frac{\partial^2 z}{\partial x^2} = -\frac{1}{x + y}$
- .

(6 marks)

- (d) A point is moving along the curve
- $z = x^2 + 3xy + 3y^2$
- with
- $x$
- decreasing at rate of
- $1.5 \text{ units s}^{-1}$
- and
- $y$
- increasing at rate of
- $2 \text{ units s}^{-1}$
- . Find the rate of change of
- $z$
- . When
- $x = 3$
- and
- $y = 1$
- .

(4 marks)

**Question 3**

- (a) The variables
- $x$
- and
- $y$
- satisfy the differential equation

$$\sqrt{x} \frac{dy}{dx} = y(2x - 3),$$

and it is given that  $y = e^2$  when  $x = 0$ . Solve the differential equation and calculate the value of  $y$  when  $x = 4$ , giving your answer correct to 3 significant figures.

(7 marks)

- (b) Find the particular solution of the differential equations

$$2xy \frac{dy}{dx} = x^2 + y^2,$$

given that  $y = 0$  when  $x = \frac{1}{2}$ .

(8 marks)

(c)  $PQRS$  is a parallelogram. Given that  $\overline{PQ} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$ ,  $\overline{PS} = \begin{pmatrix} a \\ 4 \end{pmatrix}$  and  $\overline{PR} = \begin{pmatrix} 10 \\ -b \end{pmatrix}$ , where  $a$  and  $b$  are constants.

(i) Calculate the values of  $a$  and  $b$ . (3 marks)

(ii) Find unit vector in the direction of  $\overline{PR}$ . (2 marks)

(d)

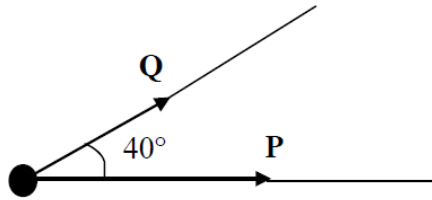


Figure 1

Two forces  $\mathbf{P}$  and  $\mathbf{Q}$ , act on a particle. The force  $\mathbf{P}$  has magnitude 7 N and the force  $\mathbf{Q}$  has magnitude 5 N. The angle between the directions of  $\mathbf{P}$  and  $\mathbf{Q}$  is  $40^\circ$ , as shown in Figure 1. The resultant of  $\mathbf{P}$  and  $\mathbf{Q}$  is  $\mathbf{F}$ .

(i) Find, to 3 decimal places, the magnitude of  $\mathbf{F}$ . (3 marks)

(ii) Find, in degrees to 3 significance figures, the angle between the directions of  $\mathbf{F}$  and  $\mathbf{P}$ . (2 marks)

**Question 4**

(a) Solve the differential equation  $4\frac{d^2y}{dx^2} + 5\frac{dy}{dx} - 9y = 0$  given that  $x = 0$  when  $y = 2$  and  $\frac{dy}{dx} = 3$ . Hence find the value of  $y$  when  $x = 2 \ln 5$ .

(10 marks)

- (b) A box contains six green balls and  $n$  blue balls.
- (i) A ball is taken out at random, the probability of taking a green ball from the box is  $\frac{2}{7}$ , find the total number of blue balls in the box.

(2 marks)

4 more green balls are added to the box and two balls are picked at random from the box.

- (ii) Find the probability of picking two green balls. (2 marks)

- (iii) Find the probability that the at least one of the balls is blue. (2 marks)

- (c) The table below shows the length of 100 fish in a pond.

Length (cm)	26-30	31-35	36-40	41-45	46-50
Number of fish	14	18	26	30	12

Find the mean, median and standard deviation for the length of the fish.

(9 marks)

**The End**

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