

**INTI**  
International College Penang

**FINAL**  
Examination Paper

(COVER PAGE)

Session : August 2018

Programme : Foundation In Science (CFSI)

Course : **MAT1211: Mathematics 2**

Date of Examination : 7 December 2018 (Friday)

Time : 2:00PM – 4:00PM Reading Time : Nil

Duration : 2 Hours

**Special Instructions** :

This paper consists of **FIVE (5)** questions. Answer **any FOUR (4)** questions in the answer booklet provided.

Materials permitted :

Non-Programmable Scientific Calculator

Materials provided :

Formula Booklet 1

Examiner(s) : **Teo Chun Yew**

Moderator : Dr. Ch'ng Pei Eng

*This paper consists of 5 printed pages, including the cover page*

INTI INTERNATIONAL COLLEGE PENANG  
 FOUNDATION IN SCIENCE (CFSI)  
 MAT 1211: MATHEMATICS 2  
 FINAL EXAM: AUGUST 2018 SESSION

**Instructions:** This paper consists of **FIVE (5)** questions. Answer any **FOUR (4)** questions in the answer booklet provided. All questions carry equal marks.

**Question 1**

(a) Given  $z = \frac{2+i}{(1-i)(1+2i)^*}$ .

- (i) Express  $z$  in  $a+ib$  form where  $a$  and  $b$  are real numbers. (4 marks)
- (ii) Find the modulus and argument of  $z$ . (2 marks)
- (iii) Use De Moivre Theorem to find  $z^4$ . Leave your answer in standard form. (4 marks)

(b) Given the following matrices:

$$\mathbf{A} = \begin{bmatrix} -2 & 4 \\ 5 & 3 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} -1 & 8 \\ 3 & 6 \end{bmatrix}.$$

Find the matrix  $\mathbf{C}$ , if  $\mathbf{C} = (\mathbf{B}^T - 2\mathbf{A})^2$ .

(4 marks)

(c) (i) Given a 3 by 3 matrix  $\mathbf{A} = \begin{bmatrix} 1 & -3 & 2 \\ 2 & 1 & -3 \\ 3 & -2 & 1 \end{bmatrix}$ , find  $\mathbf{A}^{-1}$ .

(8 marks)

(ii) Hence, or otherwise solve the following linear system

$$\begin{aligned} x - 3y + 2z &= 1 \\ 2x + y - 3z &= -5 \\ 3x - 2y + z &= 2 \end{aligned}$$

(3 marks)

**Question 2**

- (a) Use the trapezium rule with three intervals to estimate the value of

$$\int_0^{\pi/2} \sqrt{x + \cos x} \, dx,$$

giving your answer correct to 2 decimal places.

(5 marks)

- (b) Use Euler method to solve the values of  $y$  for  $x = 3(0.2)3.6$  if

$$2 \frac{dy}{dx} = e^x - e^y, \quad y(3) = 2.$$

Give your answers correct to **four (4)** decimal places.

(5 marks)

- (c) By using series expansions from formula booklet, write the first four non-zero terms of  $e^x \sin x$ .

(5 marks)

- (d) Derive  $f(x) = \sqrt[3]{1-x}$  using Maclaurin series up to and including the term in  $x^4$ . Hence, approximate the integration of  $\int_0^1 \sqrt[3]{1-x} \, dx$ , leave your answer in four decimal places.

(10 marks)

**Question 3**

- (a) Given that  $z = \ln(x^2 + y^2)$  find  $\frac{\partial^2 z}{\partial x^2}$ ,  $\frac{\partial^2 z}{\partial y^2}$  and  $\frac{\partial^2 z}{\partial x \partial y}$ .

(7 marks)

- (b) Approximate the change in the hypotenuse of a right triangle of legs 5cm and 12cm when the shorter leg is lengthened by 0.25cm and the longer leg is shortened by 0.125cm.

(8 marks)

- (c) Solve the differential equation  $2 \frac{d^2 y}{dx^2} - 3 \frac{dy}{dx} - 2y = 0$ , given that  $x = 0$  when  $y = 2$  and  $\frac{dy}{dx} = 3$ . Hence find the value of  $y$  when  $x = 2 \ln 5$ .

(10 marks)

**Question 4**

- (a) In a chemical reaction, the amount  $N$  grams of a substance after  $t$  hours is given by  $\frac{dN}{dt} = 0.005(20 - N)^2$ . If  $N = 0$  initially, find the amount of the substance 10 hours later.

(6 marks)

- (b) Solve the following differential equations:

(i)  $\frac{dy}{dx} = \frac{1 + y^3}{xy^2(1 + x^2)}$ , given that  $y = 0$  when  $x = 1$ ,

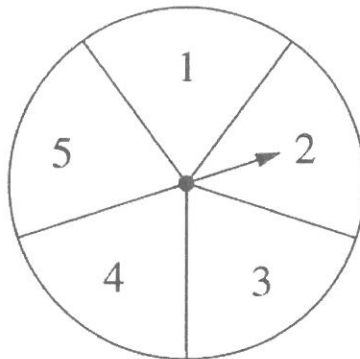
(9 marks)

(ii)  $5\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 3x^2$ .

(10 marks)

**Question 5**

- (a) A spinner is marked with the numbers 1, 2, 3, 4 and 5. When it is spun, each of the five numbers is equally likely to occur.



The spinner is spun two times and each time the score is recorded.

- (i) Event  $A$  is ‘the scores differ by 3 or more’. Find the probability of event  $A$ .
- (2 marks)
- (ii) Event  $B$  is ‘the product of the scores is less than 6’. Find the probability of event  $B$ .
- (2 marks)
- (iii) State with a reason whether events  $A$  and  $B$  are mutually exclusive.
- (2 marks)

- (b) In a survey, the percentage of meat in a certain type of take-away meal was found. The results, to the nearest integer, for 193 take-away meals are summarized in the table.

Percentage of meat	1 – 5	6 – 10	11 – 20	21 – 30	31 – 50
Frequency	59	67	38	18	11

Find the mean, median, and mode.

(9 marks)

- (c) Relative to an origin  $O$ , the position vectors of points  $A$ ,  $B$  and  $C$  are given by

$$\overrightarrow{OA} = \begin{pmatrix} 3 \\ -6 \\ p \end{pmatrix} \text{ and } \overrightarrow{OB} = \begin{pmatrix} 2 \\ -6 \\ -7 \end{pmatrix}, \overrightarrow{OC} = \frac{2}{3}\overrightarrow{OA}$$

and angle  $AOB = 90^\circ$ .

- (i) Find the value of  $p$ . (2 marks)
- (ii) Find the unit vector in the direction of  $\overrightarrow{BC}$ . (5 marks)
- (iii) Find the angle of  $ABC$  (3 marks)

**The End**

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