INTI
International College Penang

FINAL
Examination Paper

(COVER PAGE)

Session	:	April 2019
Programme	:	Foundation In Science (CFSI)
Course	:	**MAT1211: Mathematics 2**
Date of Examination	:	25 July 2019 (Thursday)
Time	:	11:00AM – 1:00PM Reading Time : Nil
Duration	:	2 Hours

Special Instructions :

This paper consists of **FIVE (5)** questions. Answer **any FOUR (4)** questions in the answer booklet provided. All questions carry equal marks.

Materials permitted :

Non-Programmable Scientific Calculator

Materials provided :

Formula Booklet 1

Examiner(s)	:	**Ms. Choong Yin Ling**
Moderator	:	Dr. Ch'ng Pei Eng

This paper consists of 5 printed pages, including the cover page

INTI INTERNATIONAL COLLEGE PENANG

FOUNDATION IN SCIENCE (CFSI)
MAT 1211 : MATHEMATICS 2
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Question 1

(a) Given that $Z_1 = -2 - i$, $Z_2 = 3 + i$ and $\overline{Z_2}$ is the complex conjugate of Z_2.

(i) Express $2Z_2 - 3Z_1$ in $a + bi$ form.

(2 marks)

(ii) Graph Z_1 in an Argand diagram. State the modulus and argument of Z_1.

(3 marks)

(iii) Use De Moivre's Theorem to find $(Z_1)^4$, write your answer in polar form.

(3 marks)

(iv) Express $\frac{\overline{Z_2}}{Z_1}$ in exponential form.

(3 marks)

(b) Find the particular solution of $\frac{dy}{dx} - 3x^2y^2 = -3x^2$ for $y(0) = 3$.

(6 marks)

(c) Find the particular solution of the following non-homogeneous differential equation.

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} - 12y = 4e^{2x} \text{ for } y(0) = 7 \text{ and } y'(0) = 0$$

(8 marks)

Question 2

(a) Use Euler's method with step size of 0.1, find the approximate value of y at $x = 0.5$ of the differential equation $\frac{dy}{dx} = e^{-x} + y$ with initial condition $y(0) = 0$. Round off your answer to 6 decimal places. Prepare a table to show your workings.

x_0	y_0	y'	Δy	y_{new}

(9 marks)

(b) Solve the following system of equations by Gaussian elimination method (row echelon form).

$$x + y + 3z = 8$$
$$x - y + 4z = 4$$
$$2x + y + 6z = 13$$

(10 marks)

(c) Find the particular solution of the differential equation $\frac{dy}{dx} = \frac{x+3y}{2x}$ given that the boundary condition is $x = 1$ when $y = 1$.

(6 marks)

Question 3

(a) Find the first three non-zero terms of Maclaurin series for xe^{-x^2}. Hence, evaluate $\int_0^1 xe^{-x^2}\,dx$ to 5 decimal places.

(9 marks)

(b) Use Cramer's rule to solve the following system of linear equations.

$$3x + y + 3z = 8$$
$$5x - 3y - 6z = 4$$
$$6x + 4y - z = 20$$

(8 marks)

(c) Find the particular solution of the given differential equation for the stated boundary conditions.

$$y'' + 6y' + 13y = 0, \text{ when } x = 0, y = 4 \text{ and } y'(0) = 0$$

(8 marks)

Question 4

(a) Let $f(x,y) = x^2 - 2xy + 2y^3$ with $x = u^2 \ln v$ and $y = 2uv^3$. Find

(i) $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$

(2 marks)

(ii) $\frac{\partial x}{\partial u}$ and $\frac{\partial y}{\partial u}$

(1 mark)

(iii) $\frac{\partial f}{\partial u}$ (in terms of u and v)

(2 marks)

(b) The height of a tree increases at a rate of 60 cm per year and the radius increases at a rate of 3 cm per year. Determine the rate of change in the volume of the tree when the tree is 6 meters high and the radius is 45 cm. (Assume the tree is a circular cylinder where $V = \pi r^2 h$)

(4 marks)

(c) Use the method of integrating factor to solve the differential equation $\frac{dy}{dx} + \frac{y}{1-x} = 1 - x^2$ subject to the boundary condition $y = 0$ when $x = -1$.

(6 marks)

(d) John's golf scores for his last 20 rounds were recorded as below.

90	106	84	103	112	100	105	81	104	98
107	95	104	108	99	101	106	102	98	101

Find the

(i) median

(2 marks)

(ii) lower quartile

(2 marks)

(iii) upper quartile

(2 marks)

(iv) mean

(2 marks)

(v) standard deviation of his scores

(2 marks)

Question 5

(a) Relative to an origin O, the position vectors of points A and B are given by

$$\overrightarrow{OA} = \begin{pmatrix} 4 \\ 1 \\ -2 \end{pmatrix} \text{ and } \overrightarrow{OB} = \begin{pmatrix} 3 \\ 2 \\ -4 \end{pmatrix}.$$

(i) Use a scalar product to find angle AOB, correct to the nearest degree.

(3 marks)

(ii) Find the unit vector in the direction of $\overrightarrow{AB}$.

(3 marks)

(iii) The position vector of the point C is given by $\overrightarrow{OC} = \begin{pmatrix} 1 \\ 4 \\ k \end{pmatrix}$, where k is a constant, and it is given that $\overrightarrow{OC} = m\overrightarrow{OA} + n\overrightarrow{OB}$, where m and n are constants. Find the values of m, n and k.

(3 marks)

(b) Catherine puts some plants in her garden. The probability that a plant will produce a flower is $\frac{7}{10}$. If there is a flower, it can only be red, yellow or purple. When there is a flower, the probability it is red is $\frac{2}{3}$ and the probability it is yellow is $\frac{1}{4}$.

(i) Draw a tree diagram to show all this information. Label the diagram and write the probabilities on each branch.

(3 marks)

(ii) A plant is chosen at random. Find the probability that it will not produce a yellow flower.

(3 marks)

(iii) If Catherine puts 120 plants in her garden, how many purple flowers would she expect?

(2 marks)

(c) Find the particular solution of the given differential equation for the stated boundary conditions.

$$6y'' + 5y' - 6y = 0, \text{ when } x = 0, y = 5 \text{ and } y'(0) = -1$$

(8 marks)

~ The End ~