INTI
International College Penang
LAUREATE INTERNATIONAL UNIVERSITIES

FINAL
Examination Paper

(COVER PAGE)

Session	:	**August 2017**
Programme	:	Foundation in Science (CFSI)
Course	:	**MAT1211: Mathematics 2**
Date of Examination	:	12 December 2017 (Tuesday)
Time	:	11:00am – 1:00pm Reading Time : Nil
Duration	:	2 Hours

Special Instructions :

This paper consists of **FIVE (5)** questions. Answer **any FOUR (4)** questions in the answer booklet provided.

Materials permitted :

Non-Programmable Scientific Calculator

Materials provided :

Formula Booklet 1

Examiner(s)	:	**Teo Chun Yew**
Moderator	:	Dr. Ch'ng Pei Eng

*This paper consists of **6** printed pages, including the cover page*

INTI INTERNATIONAL COLLEGE PENANG

FOUNDATION IN SCIENCE (CFSI)
MAT 1211: MATHEMATICS 2
FINAL EXAM: AUGUST 2017 SESSION

Instructions: This paper consists of **FIVE (5)** questions. Answer any **FOUR (4)** questions in the answer booklet provided. All questions carry equal marks.

Question 1

(a) If $z = 1 + 2i$, express each of the following in the form $a + bi$.

(i) $(iz - 1)^2$ (3 marks)

(ii) $\dfrac{z}{4 - z^2}$ (4 marks)

(b) Solve the equation $z^2 = -5 + 12i$ for all the complex roots, z. (6 marks)

(c) Given that $x^2 + 4y^2 - 2z^2 - 4 = 0$.

(i) Express the equation of the curve in the form $\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 - \left(\frac{z}{c}\right)^2 = 1$ where a, b, and c are integers.

(2 marks)

(ii) Hence, identify the name of the curve.

(1 mark)

(d) Derive $f(x) = e^{-x} \sin x$ using Maclaurin series up to and including the term in x^3. Hence, approximate $\int_0^{0.5} e^{-x} \sin x dx$ correct to **four (4)** decimal places.

(9 marks)

Question 2

(a) Given a matrix **A** as follow, find $\mathbf{A}^{-1}$.

$$\mathbf{A} = \begin{bmatrix} 2 & -1 & 1 \\ 1 & 0 & 1 \\ 3 & -1 & 4 \end{bmatrix}$$

(7 marks)

(b) Given that the system of linear equations is

$$\begin{aligned} 2x + y - 3z &= 1 \\ 3x - y - 4z &= 7 \\ 5x + 2y - 6z &= 5 \end{aligned} .$$

By using Cramer's Rule, find the value of y and z.

(7 marks)

(c) Use Euler's method to solve the values of y for $x = 1(0.1)1.3$ if

$$x\frac{dy}{dx} = y^2 - 3x^2, \; y(1) = 2 .$$

Give your answers correct to **four (4)** decimal places.

(6 marks)

(d) Using the trapezoidal rule with 4 subintervals, to approximate area under the curve $y = xe^x$ between $x = 1$ and $x = 3$. Give your answer correct to three decimal places.

(5 marks)

Question 3

(a) Given that $z = e^{x+y}\sin(2x+3y)$, find $\dfrac{\partial^2 z}{\partial x^2}$, $\dfrac{\partial^2 z}{\partial y^2}$ and $\dfrac{\partial^2 z}{\partial y \partial x}$.

(7 marks)

(b) If $z = \dfrac{x-y}{x+y}$ and $x = s^2 + t^2$, $y = 2st$, find $\dfrac{\partial z}{\partial s}$ and $\dfrac{\partial z}{\partial t}$.

(6 marks)

(c) The total resistance R of 2 resistors (x, y) connected in parallel is

$$R = \frac{xy}{x+y}.$$

Suppose x and y are measured to be 200 Ω (ohms) and 400 Ω, respectively. If x increases by 1 Ω and y decreases by 4 Ω, estimate the change in R using the total differential.

(6 marks)

(d) The base of radius r of a right circular cone is decreasing at the rate of 2.0 mm s^{-1} while the height h is increasing at 3.0 mm s^{-1}. Use partial differentiation to estimate the rate at which the volume V is changing when $r = 6$ mm and $h = 3$ mm.

(6 marks)

Question 4

(a) Given a differential equation $xy\frac{dy}{dx} = 2x^2 - y^2$.

(i) By using the substitution $y = vx$, where v is a function of x, show that the differential equation can be reduced to the form $x\frac{dv}{dx} = \frac{2-2v^2}{v}$.

(4 marks)

(ii) Hence, find the particular solution of the differential equation such that when $x = 2$, $y = 1$.

(6 marks)

(b) Given a differential equation $\frac{dy}{dx} + y\cot x = \sin 3x$.

(i) Find the integrating factor of the differential equation. (3 marks)

(ii) Find the particular solution of the differential equation such that when $y = 0$, $x = \frac{\pi}{4}$.

(5 marks)

(c) Solve the differential equation $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 2x^2 - 3$. (7 marks)

Question 5

(a) In a game, a turn involves rolling two dice, each with faces marked 0, 1, 2, 3, 4, and 5. The score for each turn is calculated by multiplying the two numbers uppermost on the dice.

(i) What is the probability of scoring zero on the first turn? (2 marks)

(ii) What is the probability of scoring 16 or more on the first turn? (2 marks)

(iii) What is the probability that the sum of the scores in the first two turns is less than 45 ?

(4 marks)

(b) The following table shows the age distribution of blood donors.

Age (years)	15 - 24	25 - 34	35 - 44	45 - 54	55 - 56
Frequency	26	30	42	34	18

Find the

(i) mean, and (3 marks)

(ii) mode. (4 marks)

(c) Relative to an origin O, the position vectors of points A, B, and C are given by

$$\overline{OA} = 2\underset{\sim}{i} + \underset{\sim}{j} - 2\underset{\sim}{k}\,,\ \overline{OB} = 5\underset{\sim}{i} - \underset{\sim}{j} + a\underset{\sim}{k}\,,\ \text{and}\ \overline{OC} = 2\underset{\sim}{i} + 6\underset{\sim}{j} - 3\underset{\sim}{k}$$

respectively, where a is a constant.

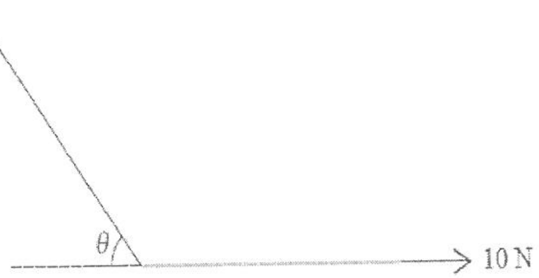
(i) Find the value of a in the case where angle $AOB = 90°$.

(2 marks)

(ii) Find the possible values of a for which the lengths of AB and OC are equal.

(5 marks)

(d)



Forces of magnitudes 10 N and 8 N act in directions as shown in the diagram. The resultant of the two forces has magnitude 8 N. Show that $\cos\theta = \frac{5}{8}$.

(3 marks)

The End

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