INTI International College Penang

FINAL
ALTERNATIVE ASSESSMENT

(COVER PAGE)

Session	:	**August 2021**
Programme	:	American Degree Transfer Program (AUP)
Course	:	**MAT216: Introduction To Linear Algebra, Lab**
Date of Examination	:	10 December 2021 (Friday)
Time	:	8.00am – 10.15am Reading Time : Nil
Duration	:	2 Hours 15 Minutes

Special Instructions :

This paper consists of **FOUR (4)** questions. Answer ALL **FOUR (4)** questions. Show your working clearly.

Material permitted :

Non-Programmable Scientific Calculator

Materials provided :

Nil

Examiner(s)	:	**Nhivashni Womasuthan**
Chief Moderator	:	Chow Seng Chu

*This paper consists of **4** printed pages, including the cover page*

AMERICAN DEGREE TRANSFER PROGRAM (AUP)
MAT216: INTRODUCTION TO LINEAR ALGEBRA, LAB
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Question 1

Given the following linear system:

$$x + (\lambda + 1)y + z = \mu^2$$
$$\lambda x + 2y + z = 1$$
$$2x + 3y + z = 3$$

(a) Write the augmented matrix $[A|\ \underline{b}]$ for the above linear system. (2 marks)

(b) Determine all possible values of λ and μ so that the linear system has a unique solution. (8 marks)

(c) Find all the corresponding values for λ and μ so that the linear system will have no solution and infinitely many solutions, respectively. (10 marks)

(d) Find the general solution of the linear system if $\lambda = 1$ and $\mu = 1$. (5 marks)

Question 2

(a) Let $W = \left\{\underline{p} = p(x) = a + bx + cx^2 | a + b + c = 0\right\} \subseteq P_2$, where P_2 is the set of all polynomials of degree two or less.

(i) Is W closed under ordinary vector addition? Justify your answer. (3 marks)

(ii) Is W closed under ordinary scalar multiplication on vectors? Justify your answer.

(2 marks)

(iii) Is W a real subspace of P_2? Justify your answer. (1 mark)

(b) Let $A = \begin{bmatrix} 1 & 4 & 5 & 6 & 9 \\ 3 & -2 & 1 & 4 & -1 \\ -1 & 0 & -1 & -2 & -1 \\ 2 & 3 & 5 & 7 & 8 \end{bmatrix}$. Determine a basis, each of which, for

(i) the row space of A, (5 marks)

(ii) the column space of A, and (3 marks)

(iii) the null space of A. (6 marks)

(c) Determine whether the following function is a linear transformation.

$$T: R^2 \rightarrow R^2 \text{ with } T\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x+y \\ y \end{bmatrix}$$ (5 marks)

Question 3

Given the matrix

$$A = \begin{bmatrix} 1 & 2 & 5 \\ 0 & -5 & 0 \\ 4 & -2 & 0 \end{bmatrix}$$

(a) Is A orthogonally diagonolizable? Why? (2 marks)

(b) Find all eigenvalues and all the corresponding eigenvectors of A. (16 marks)

(c) From the results obtained in (b), state a diagonalizer P and its inverse P^{-1} for A. (4 marks)

(d) Hence, obtain the second row of A^3. (3 marks)

Question 4

(a) Let $\Re^3$ have the following inner product $< , >$:

$$\langle \underline{u}, \underline{v} \rangle = u_1 v_1 + 2u_2 v_2 + 3u_3 v_3 \text{ where } \underline{u} = (u_1, u_2, u_3) \text{ and } \underline{v} = (v_1, v_2, v_3)$$

(i) Apply the Gram-Schmidt process to transform the basis

$V = \{\underline{v_1} = (1,1,1), \underline{v_2} = (1,1,0), \underline{v_3} = (1,0,0)\}$ into an orthogonal basis

$S_1 = \{\underline{w_1}, \underline{w_2}, \underline{w_3}\}$. (7 marks)

(ii) Normalize the orthogonal basis S_1 to obtain the orthonormal basis $S_2 = \{\underline{b_1}, \underline{b_2}, \underline{b_3}\}$.

(4 marks)

(b) Let $T: \Re^2 \to \Re^3$ be a linear transformation defined by

$$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x + 2y \\ -x \\ y \end{bmatrix} \text{ for all } \begin{bmatrix} x \\ y \end{bmatrix} \in \Re^2.$$

Let $B = \left\{ \underline{u_1} = \begin{bmatrix} 1 \\ 2 \end{bmatrix},\ \underline{u_2} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}, \underline{u_3} = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} \right\}$ and

$B' = \left\{ \underline{v_1} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix},\ \underline{v_2} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \underline{v_3} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}$ be the bases of $\Re^2$ and $\Re^3$.

(i) Find a basis for the range of T. (4 marks)

(ii) Find a basis for the kernel of T. (5 marks)

(iii) Find the matrix $[T]_{B',B}$ for T with respect to B and B' in $\Re^3$. (5 marks)

~THE END~